

## Footnotes

1. I probably mean “a functor from some fixed “structure multi-category” to the multi-category of sets, extended to formal linear combinations”.
2. A Leibniz algebra is a Lie algebra minus the anti-symmetry of the bracket; I have previously erroneously asserted that here  $\mathcal{A}(K)$  is Lie; however see the comment by Conant attached to this talk’s video page.
3. See my paper [BN1] and my talk/handout/video [BN3].
4. See [BN5] and my talk/handout/video [BN4].
5. Not so old and not quite written up. Yet see [BN2].

## References

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- [BN4] D. Bar-Natan, *From the  $ax + b$  Lie Algebra to the Alexander Polynomial and Beyond*, talk given in Chicago on September 11, 2010, <http://www.math.toronto.edu/~drorbn/Talks/Chicago-1009/>.
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## Plan

1. (8 minutes) The Peter Lee setup for  $(K, I)$ , “all interesting graded equations arise in this way”.
2. (3 minutes) Example: the pure braid group (mention  $PvB$ , too).
3. (3 minutes) Generalized algebraic structures.
4. (1 minute) Example: quandles.
5. (4 minutes) Example: parenthesized braids and horizontal associators.
6. (6 minutes) Example: KTGs and non-horizontal associators. (“Bracket rise” arises here).
7. (8 minutes) Example:  $wKO$ ’s and the Kashiwara-Vergne equations.
8. (12 minutes)  $vKO$ ’s, bi-algebras, E-K, what would it mean to find an expansion, why I care (stronger invariant, more interesting quotients).
9. (5 minutes)  $wKO$ ’s,  $uKO$ ’s, and Alekseev-Enriquez-Torossian.