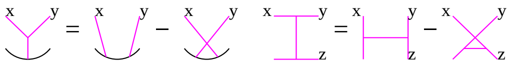
Cosmic Coincidences and Several Other Stories, 2

Dror Bar−Natan at the University of Tennessee
March 4, 2011, http://www.math.toronto.edu/~drorbn/Talks/Tennessee−1103/

"Low Algebra" and universal formulae in Lie algebras.



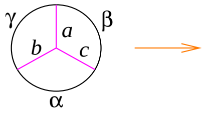
[x,y] = xy − yx [[x,y],z]=[x,[y,z]]−[y,[x,z]]


Sophus Lie

More precisely, let $\mathfrak{g} = \langle X_a\rangle$ be a Lie algebra with an orthonormal basis, and let $R = \langle v_\alpha\rangle$ be a representation. Set

$$f_{abc} := \langle [X_a, X_c], X_c\rangle \qquad X_a v_\beta = \sum_\gamma r^\beta_{a\gamma} v_\gamma$$

and then

$W_{\mathfrak{g},R}$:


$$\sum_{abc\alpha\beta\gamma} f_{abc} r^\beta_{a\gamma} r^\gamma_{b\alpha} r^\alpha_{c\beta}$$

$W_{\mathfrak{g},R}\circ Z$ is often interesting:

$\mathfrak{g} = sl(2) \longrightarrow$

The Jones polynomial

$\mathfrak{g} = sl(N) \longrightarrow$

Przytycki
The HOMFLYPT polynomial

$\mathfrak{g} = so(N) \longrightarrow$

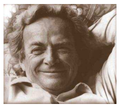
The Kauffman polynomial

Chern-Simons-Witten theory and Feynman diagrams.

$$\int_{\mathfrak{g}\text{-connections}} \mathcal{D}A\, hol_K(A) \exp\left[\frac{ik}{4\pi}\int_{\mathbb{R}^3} \mathrm{tr}\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right)\right]$$

$$\longrightarrow \sum_{D:\text{ Feynman diagram}} W_{\mathfrak{g}}(D) \sum\!\!\!\!\!\!\int \mathcal{E}(D) \longrightarrow \sum_{D:\text{ Feynman diagram}} D \sum\!\!\!\!\!\!\int \mathcal{E}(D)$$


Witten


Feynman

Definition. V is finite type (Vassiliev, Goussarov) if it vanishes on sufficiently large alternations as on the right

Theorem. All knot polynomials (Conway, Jones, etc.) are of finite type.

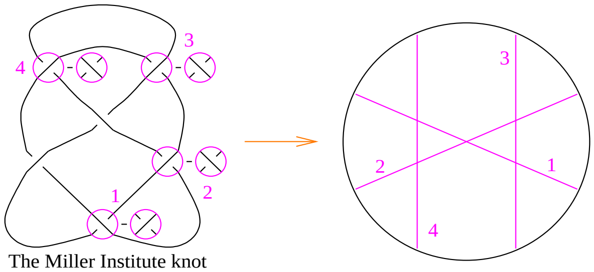
Conjecture. (Taylor's theorem) Finite type invariants separate knots.

Theorem. *Z(K)* is a universal finite type invariant!

(sketch: to dance in many parties, you need many feet).


Vassiliev


Goussarov

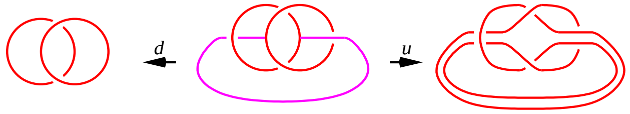


The Miller Institute knot

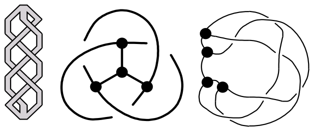
Knots are the wrong objects to study in knot theory!

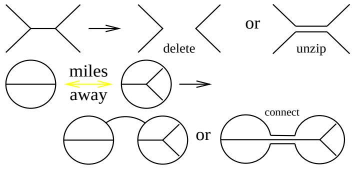
They are not finitely generated and they carry no interesting operations.

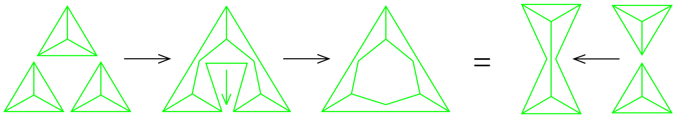
Algebraic Knot Theory



Knotted Trivalent Graphs







Theorem ($\sim$, "High Algebra"). A homomorphic Z is the same as a "Drinfel'd Associator".


Drinfel'd

The u→v→w & p Stories

explained sketched could explain could explain, gaps remain more gaps then explains mystery

	Topology	Combinatorics	Low Algebra	High Algebra	Counting Coincidences Conf. Space Integrals	Quantum Field Theory	Graph Homology
u-Knots ↓	The usual Knotted Objects (KOs) in 3D — braids, knots, links, tangles, knotted graphs, etc.	Chord diagrams and Jacobi diagrams, modulo $4T$, STU, IHX, etc.	Finite dimensional metrized Lie algebras, representations, and associated spaces.	The Drinfel'd theory of associators.	Today's work. Not beautifully written, and some detour-forcing cracks remain.	Perturbative Chern-Simons-Witten theory.	The "original" graph homology.
v-Knots ↓	Virtual KOs — "algebraic", "not embedded"; KOs drawn on a surface, mod stabilization.	Arrow diagrams and v-Jacobi diagrams, modulo $6T$ and various "directed" STUs and IHXs, etc.	Finite dimensional Lie bi-algebras, representations, and associated spaces.	Likely, quantum groups and the Etingof-Kazhdan theory of quantization of Lie bi-algebras.	No clue.	No clue.	No clue.
w-Knots	Ribbon 2D KOs in 4D; "flying rings". Like v, but also with "overcrossings commute".	Like v, but also with "tails commute". Only "two in one out" internal vertices.	Finite dimensional co-commutative Lie bi-algebras ($\mathfrak{g}\ltimes\mathfrak{g}^*$), representations, and associated spaces.	The Kashiwara-Vergne-Alekseev-Torossian theory of convolutions on Lie groups / algebras.	No clue.	Probably related to 4D BF theory.	Studied.
p-Objects	No clue.	"Acrobat towers" with 2-in many-out vertices.	Poisson structures.	Deformation quantization of poisson manifolds.	Configuration space integrals are key, but they don't reduce to counting.	 Work of Cattaneo.	Studied. Hyperbolic geometry ?

Video and more at `http://www.math.toronto.edu/~drorbn/Talks/Tennessee-1103/` and at `http://www.math.toronto.edu/~drorbn/Talks/Caen-1206/#Colloquium`