

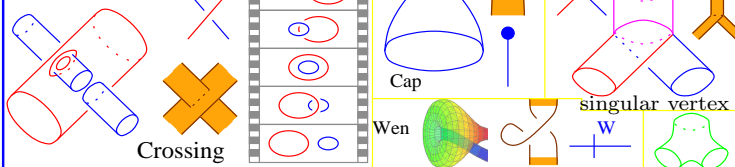
## 2. w-Knots, Alekseev–Torossian, and baby Etingof–Kazhdan

I understand Drinfel'd and Alekseev–Torossian, I don't understand Etingof–Kazhdan yet, and I'm clueless about Kontsevich  
 Dror Bar–Natan, Montpellier, June 2010, <http://www.math.toronto.edu/~drorbn/Talks/Montpellier-1006/>

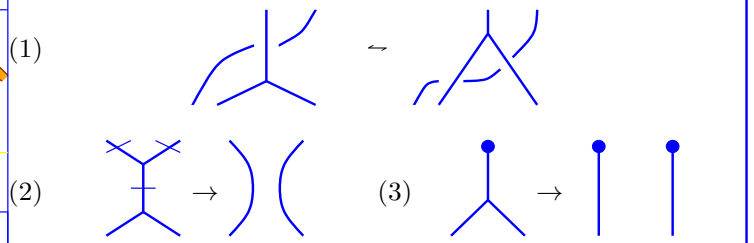
### Trivalent w-Tangles.

$$wTT = CA \left\langle \begin{array}{c|c|c} w\text{-} & w\text{-} & \text{unary } w\text{-} \\ \text{generators} & \text{relations} & \text{operations} \end{array} \right\rangle$$

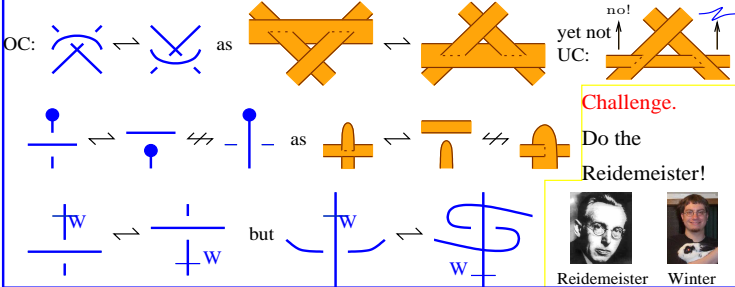
#### The w-generators.



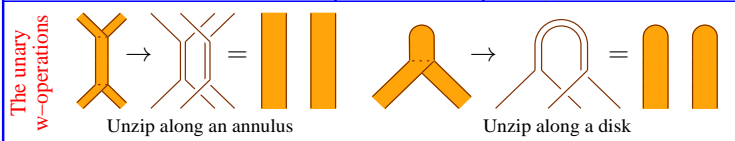
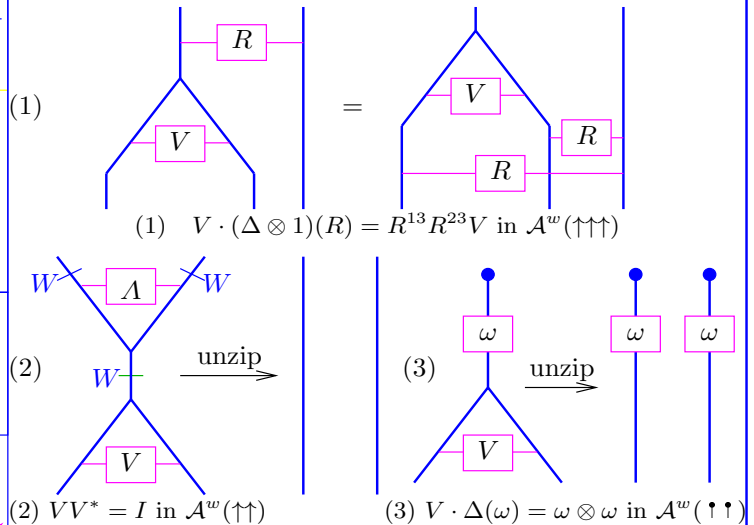
**Knot-Theoretic statement.** There exists a homomorphic expansion  $Z$  for trivalent  $w$ -tangles. In particular,  $Z$  should respect  $R4$  and intertwine annulus and disk unzips:



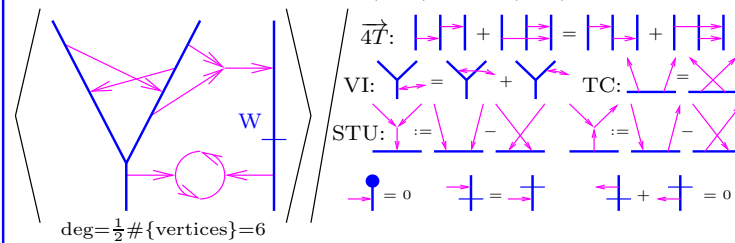
The  $w$ -relations include  $R234$ ,  $VR1234$ ,  $D$ , Overcrossings Commute ( $OC$ ) but not  $UC$ ,  $W^2 = 1$ , and funny interactions between the wen and the cap and over- and under-crossings:



**Diagrammatic statement.** Let  $R = \exp \uparrow \in \mathcal{A}^w(\uparrow\uparrow)$ . There exist  $\omega \in \mathcal{A}^w(\uparrow)$  and  $V \in \mathcal{A}^w(\uparrow\uparrow)$  so that



#### w-Jacobi diagrams and $\mathcal{A}$ . $\mathcal{A}^w(Y \uparrow) \cong \mathcal{A}^w(\uparrow\uparrow)$ is

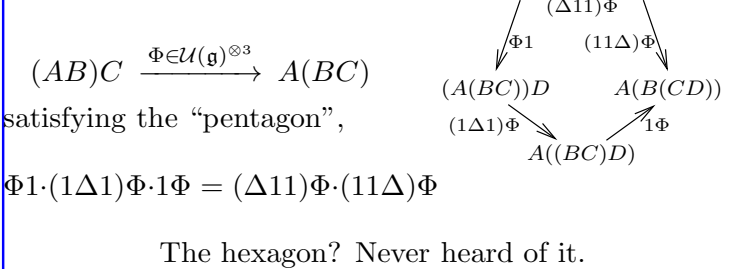


**Alekseev-Torossian statement.** There are elements  $F \in \text{TAut}_2$  and  $a \in \mathfrak{t}_1$  such that

$$F(x+y) = \log e^x e^y \quad \text{and} \quad jF = a(x) + a(y) - a(\log e^x e^y).$$

**Theorem.** The Alekseev-Torossian statement is equivalent to the knot-theoretic statement.

#### An Associator:



**Proof.** Write  $V = e^c e^{uD}$  with  $c \in \mathfrak{t}_2$ ,  $D \in \mathfrak{tder}_2$ , and  $\omega = e^b$  with  $b \in \mathfrak{t}_1$ . Then (1)  $\Leftrightarrow e^{uD}(x+y)e^{-uD} = \log e^x e^y$ , (2)  $\Leftrightarrow I = e^c e^{uD}(e^{uD})^* e^c = e^{2c} e^{jD}$ , and (3)  $\Leftrightarrow e^c e^{uD} e^{b(x+y)} = e^{b(x)+b(y)} \Leftrightarrow e^c e^{b(\log e^x e^y)} = e^{b(x)+b(y)} \Leftrightarrow c = b(x) + b(y) - b(\log e^x e^y)$ .

**The Alekseev-Torossian Correspondence.**  $\{\text{Drinfel'd Associators}\} \Leftrightarrow \{\text{Solutions of KV}\}$ .

We need an even bigger algebraic structure!  
 (green knotted trivalent graphs in  $\mathbb{R}^3$  ( $u$ ))  $\xrightarrow{\alpha_e}$  (blue tubes and red strings in  $\mathbb{R}^4$  ( $\bar{w}$ ))

