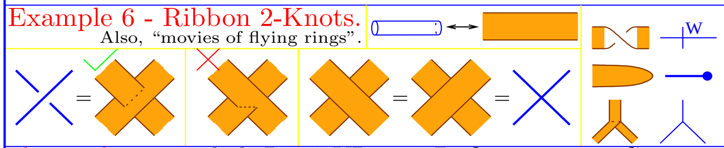
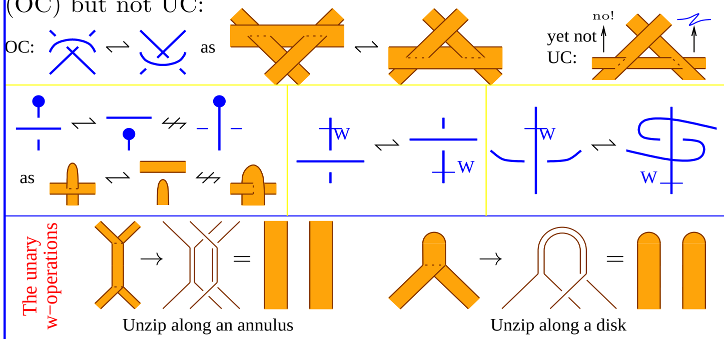
Facts and Dreams About v−Knots and Etingof−Kazhdan, 2

Example 6 - Ribbon 2-Knots. Also, "movies of flying rings".



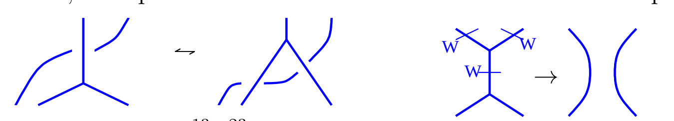
The w-relations include R234, VR1234, D, Overcrossings Commute (OC) but not UC:



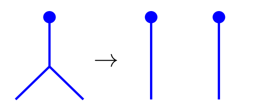
Trivalent w-Tangles. 30

$$\text{wTT} = \text{PA}\left\langle \begin{matrix}\text{w-}\\ \text{generators}\end{matrix} \middle| \begin{matrix}\text{w-}\\ \text{relations}\end{matrix} \middle| \begin{matrix}\text{unary w-}\\ \text{operations}\end{matrix}\right\rangle = \text{CA}\left\langle\begin{matrix}\text{same}\\ \text{w/o}\times\end{matrix}\right\rangle$$

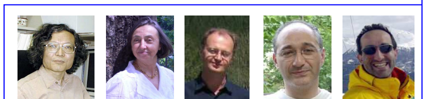
Theorem. There exists a homomorphic expansion Z for wTT. In particular, Z respects $R4$ and intertwines annulus and disk unzips:



$V\cdot(\Delta\otimes 1)(R) = R^{13}R^{23}V$ in $\mathcal{A}^w(\uparrow_3)$ $\qquad VV^* = I$ in $\mathcal{A}^w(\uparrow_2)$



$V\cdot\Delta(\omega) = \omega\otimes\omega$ in $\mathcal{A}^w(\uparrow_2)$



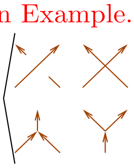
Kashiwara-Vergne-Alekseev-Enriquez-Torrosian

Alekseev-Torossian [AT] (equivalent to Kashiwara-Vergne [KV]). There are elements $F\in \text{TAut}_2$ and $a\in\mathfrak{tr}_1$ such that

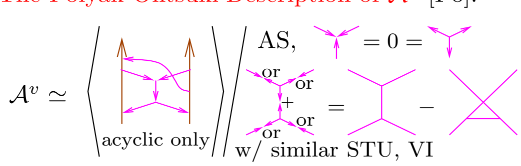
$$F(x+y) = \log e^x e^y \quad\text{and}\quad jF = a(x)+a(y)-a(\log e^x e^y).$$ 33

Theorem. That's equivalent to a homomorphic expansion for wTT.

The Main Example.

vTT= PA⟨  | R234, VR234, D, yet not UC, OC | unzips ⟩ $= \widetilde{\text{CA}}\left\langle\begin{matrix}\text{same}\\ \text{w/o}\times\end{matrix}\right\rangle$

The Polyak-Ohtsuki Description of $\mathcal{A}^v$ [Po].



Polyak

Ohtsuki

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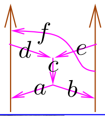
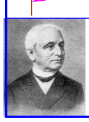
$\mathcal{A}^v$ pairs with Lie bialgebras. Let $\mathfrak{g}_+$ be a Lie bialgebra with basis X_a, bracket $[\cdot,\cdot]$, cobracket δ, dual $\mathfrak{g}_- = \mathfrak{g}_+^\star$, dual basis X^a for $\mathfrak{g}_-$, double $\mathfrak{g}=\mathfrak{g}_+\oplus\mathfrak{g}_-$, structure constants $[X_a,X_b]=\sum b_{ab}^c X_c$ and co-structure constants $\delta(X_a)=\sum c_a^{bc}X_b\otimes X_c$. Then

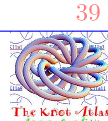


$$\longrightarrow \sum_{a,b,c,d,e,f=1}^{\dim\mathfrak{g}} b_{de}^c c_c^{ba} X_a X^d X_f \otimes X_b X^f X^e \in \mathcal{U}(\mathfrak{g})^{\otimes 2}$$ 39



"God created the knots, all else in topology is the work of mortals."
Leopold Kronecker (modified)

www.katlas.org



Forbidden Theorem [EK, Ha, ?]. There exists a homomorphic expansion Z for vTT.

Why Forbidden (to me)?

- Minor statement details may be off.
- No fully written proof.
- I don't understand the proof.
- There isn't yet a knot-theoretic view of the proof, like there is in the w-case. 42

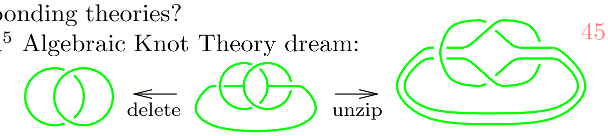
Etingof

Haviv

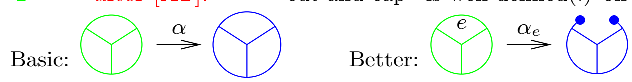
Kazhdan

Why Should We Care?

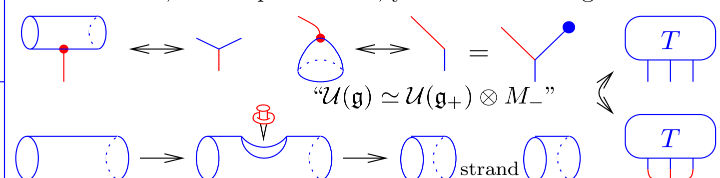
- A gateway into the forbidden territory of "quantum groups".
- Abstractly more pleasing: We study the things, and not just their representations.
- $\mathcal{A}^v$ is sometimes easier than $\mathcal{A}^u$: Alexander, say, arises easily from the 2D Lie algebra[4].
- Potentially, $\mathcal{A}^v$ has many more "internal quotients" than there are Lie bialgebras. What are they and what are the corresponding theories?
- My old[5] Algebraic Knot Theory dream: 45

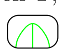
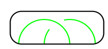


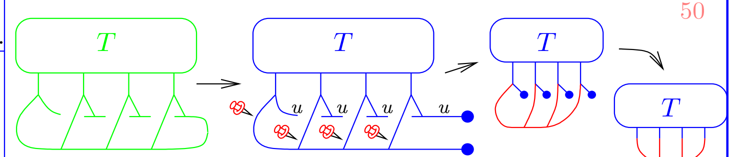
$V \to \Phi^{1\text{-loop}}$ after [AT]. "cut and cap" is well-defined(!) on $\mathcal{K}^u$

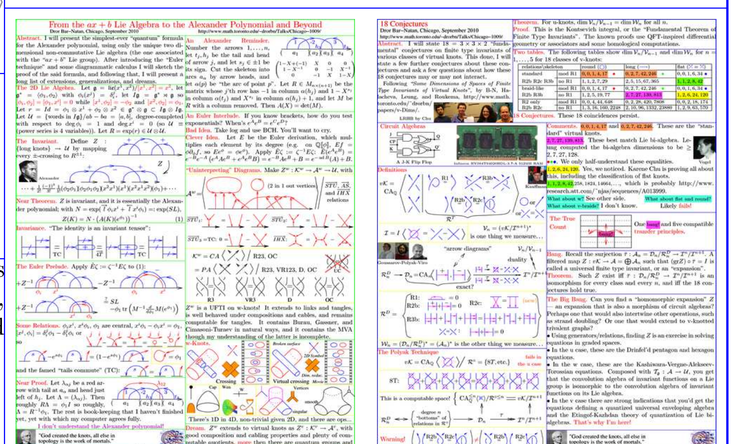


$\Phi \to V$ after [AET]. In $\mathcal{K}^{\overline{w}}$ allow tubes and strands and tube-strand vertices, allow "punctures", yet allow no "tangles".



The generators of $\mathcal{K}^{\overline{w}}$ can be written in terms of the generators of $\mathcal{K}^u$ (i.e., given Φ, can write a formula for V). With T any classical tangle, esp.  or , consider the "sled" 50





Alexander is easy! In Chicago, [BN4] Many kinds of virtuals!

Help Needed!

Video and more at http://www.math.toronto.edu/~drorbn/Talks/SwissKnots-1105/