## Day 1 - u, v, w: topology and philosophy u, v, and w-Knots: Topology, Combinatorics and Low and High Algebra Dror Bar-Natan, Goettingen, April 2010 http://www.math.toronto.edu/~drorbn/Talks/Goettingen-1004/ Plans and Dreams "An Algebraic Structure" arbitrary algebraic projectivization 'a problem in` $\mathcal{O} =$ graded algebra structure $\mathcal{O}_2$ $\mathcal{O}_1$ Feed knot-things, get Lie algebra things. Feed u-knots, get Drinfel'd associators. $\psi_4$ Feed w-knots, get Kashiware-Vergne-Alekseev-Torossian. objects of $\psi_3$ Dream: Feed v-knots, get Etingof-Kazhdan. $\mathcal{O}_3$ $\mathcal{O}_4$ kind 3 Dream: Knowing the question whose answer is 42, or E-K, will be useful to algebra and topology. • Has kinds, objects, operations, and maybe constants. Perhaps subject to some axioms. We always allow formal linear combinations. Homomorphic expansions for a filtered algebraic structure $\mathcal{K}$ : $ops \hookrightarrow \mathcal{K} = \mathcal{K}_0 \supset \mathcal{K}_1 \supset \mathcal{K}_2 \supset \mathcal{K}_3 \supset \dots$ $\mathrm{ops} \ ^{\subset} \mathrm{gr} \ \mathcal{K} := \mathcal{K}_0/\mathcal{K}_1 \ \oplus \ \mathcal{K}_1/\mathcal{K}_2 \ \oplus \ \mathcal{K}_2/\mathcal{K}_3 \ \oplus \ \mathcal{K}_3/\mathcal{K}_4 \ \oplus \ \dots$ 1-KnotAn expansion is a filtration respecting $Z: \mathcal{K} \to \operatorname{gr} \mathcal{K}$ that ( knots R123: $\langle \rangle = \rangle$ , $\langle \rangle = \rangle$ "covers" the identity on $\operatorname{gr} \mathcal{K}$ . A homomorphic expansion is l &links f an expansion that respects all relevant "extra" operations. Circuit Algebras The set of all K =Just for fun. b/w 2D projections of reality CP $K/K_1 \leftarrow K/K_2 \leftarrow K/K_3 \leftarrow K/K_4 \leftarrow$ A J-K Flip Flop Crop Infineon HYS64T64020HDL-3.7-A 512MB RAM Rotate (CA :=Circuit Algebra) v-knots $= CA \langle \rangle$ Adjoin An expansion Z is a choice of a 'progressive scan" algorithm. crop $\mathcal{K}/\mathcal{K}_1 \oplus \mathcal{K}_1/\mathcal{K}_2 \oplus \mathcal{K}_2/\mathcal{K}_3 \oplus \mathcal{K}_3/\mathcal{K}_4 \oplus \mathcal{K}_4/\mathcal{K}_5 \oplus \mathcal{K}_5/\mathcal{K}_6 \oplus \cdots$ rotate adjoin $\ker(\mathcal{K}/\mathcal{K}_4 \rightarrow \mathcal{K}/\mathcal{K}_3)$ Filtered algebraic structures are cheap and plenty. w-Tangles $\mathcal{K}$ , allow formal linear combinations, let $\mathcal{K}_1 = \mathcal{I}$ be the ideal generated by differences (the "augmentation ideal"), and let $\{w\text{-Tangles}\} = v\text{-Tangles}$ $\mathcal{K}_m := \langle (\mathcal{K}_1)^m \rangle$ (using all available "products"). Examples. 1. The projectivization of a group is a graded The w-generators Broken surface associative algebra. 2. Quandle: a set Q with an op $\wedge$ s.t. 2D Symbol $1 \wedge x = 1, \quad x \wedge 1 = x,$ (appetizers) $(x \wedge y) \wedge z = (x \wedge z) \wedge (y \wedge z).$ (main) proj Q is a graded Leibniz algebra: Roughly, set $\bar{v} := (v-1)$ Dim. reduc. (these generate I!), feed $1 + \bar{x}$ , $1 + \bar{y}$ , $1 + \bar{z}$ in (main), collect Virtual crossing Movie Crossing the surviving terms of lowest degree: A Ribbon 2-Knot is a surface S embedded in $\mathbb{R}^4$ that bounds $(\bar{x} \wedge \bar{y}) \wedge \bar{z} = (\bar{x} \wedge \bar{z}) \wedge \bar{y} + \bar{x} \wedge (\bar{y} \wedge \bar{z}).$ an immersed handlebody B, with only "ribbon singularities"; a ribbon singularity is a disk D of trasverse double points, $\overline{\text{Our case(s)}}$ . whose preimages in B are a disk $D_1$ in the interior of B and Z: high algebra algebra $\mathfrak g$ " $\mathcal{U}(\mathfrak{g})$ " a disk $D_2$ with $D_2 \cap \partial B = \partial D_2$ , modulo isotopies of S alone. $\operatorname{proj} \mathcal{K}$ solving finitely many equations in finitely low algebra: pic represent many unknowns formulas $\mathcal{K}$ is knot theory or topology; proj $\mathcal{K} = \bigoplus \mathcal{I}^m/\mathcal{I}^{m+1}$ is finite combinatorics: bounded-complexity diagrams modulo simple The w-relations include R234, VR1234, M, Overcrossings relations. Commute (OC) but not UC: 'God created the knots, all else in vet not ∤ topology is the work of mortals.' Leopold Kronecker (modified) www.katlas.org Also see http://www.math.toronto.edu/~drorbn/papers/WKO/