

Dror Bar-Natan: Classes: 2002-03: Math 157 - Analysis I:

## Homework Assignment 15

Assigned Tuesday January 14; due Friday January 24, 2PM at SS 1071

web version: <http://www.math.toronto.edu/~drorbn/classes/0203/157AnalysisI/HW15/HW15.html>

### Required reading

All of Spivak Chapter 15.

### To be handed in

From Spivak Chapter 15: 2 (even parts), 4 (even parts), 9, 15, 18.

### Recommended for extra practice

From Spivak Chapter 15: 2 (odd parts), 4 (odd parts), 7, 14, 27.

### An aside

Here's a short Mathematica session that computes an approximation of  $2 \int_{-1}^1 \sqrt{1-x^2} dx$ :

```
drorbn@coxeter:~/classes/157AnalysisI:1 math
Mathematica 4.1 for IBM AIX
Copyright 1988-2000 Wolfram Research, Inc.
```

```
In[1]:= n = 1000; t[i_] := -1. + 2i/1000; f[x_] := Sqrt[1 - x^2]
```

```
In[2]:= 2 * Sum[f[t[i]]*(t[i] - t[i - 1]), {i, 1, 1000}]
```

```
Out[2]= 3.14149
```

### Just for fun

Compute the limit

$$\lim_{N \rightarrow \infty} \cos x \cdot \cos \frac{x}{2} \cdot \cos \frac{x}{4} \cdot \cos \frac{x}{8} \cdots \cos \frac{x}{2^N}$$