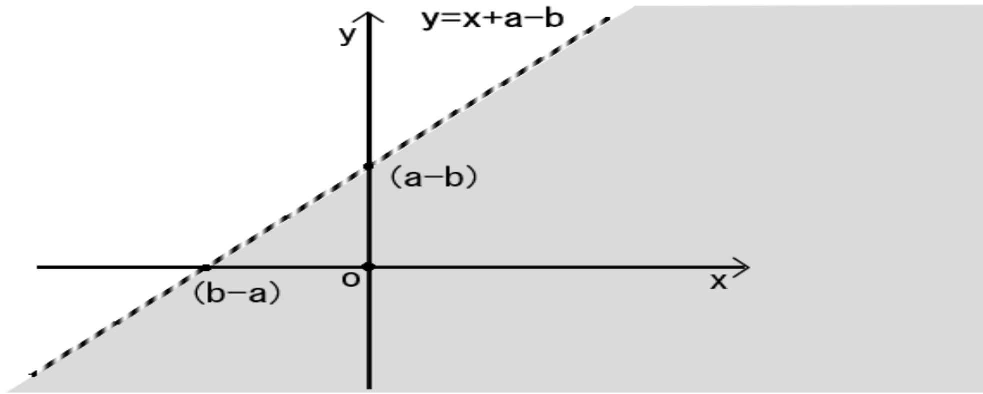
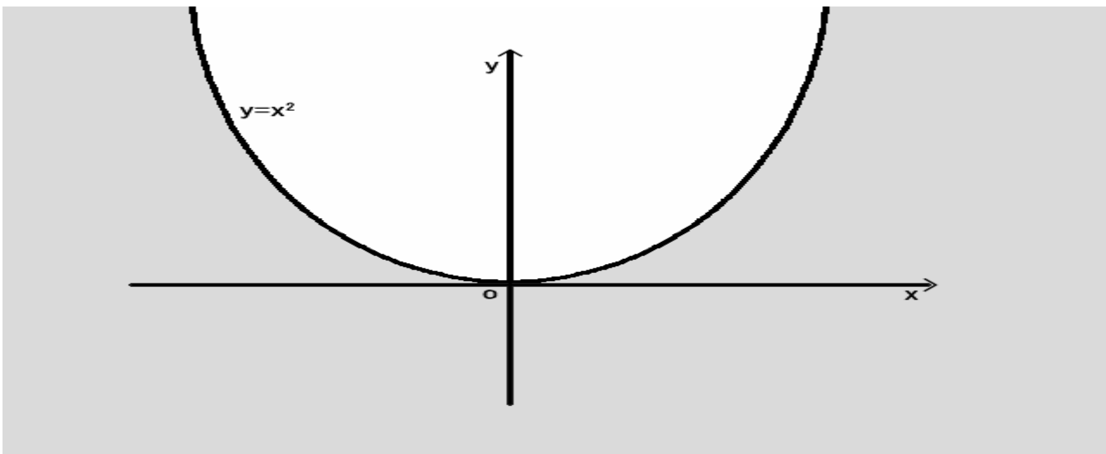
Chapter 4

3. Draw the set of all point (x,y) satisfying the following conditions.

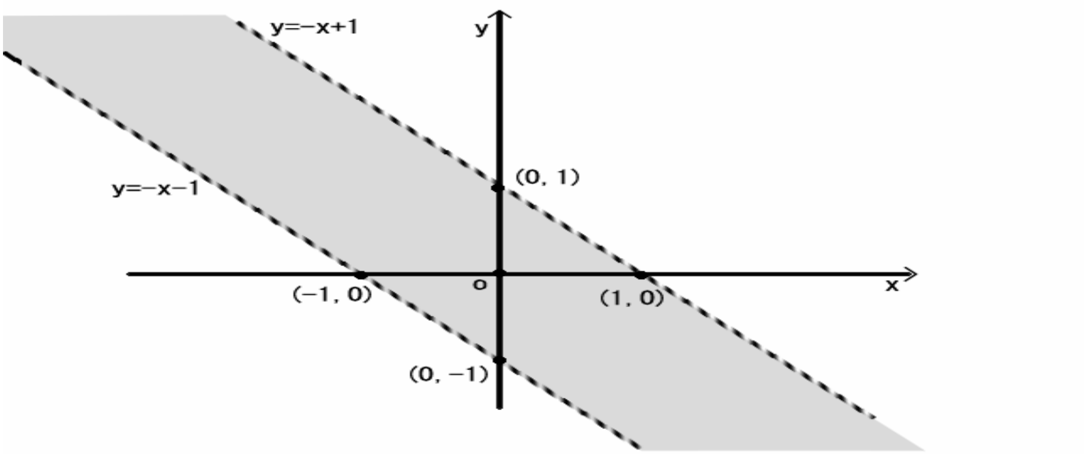
ii) $x+a>y+b$



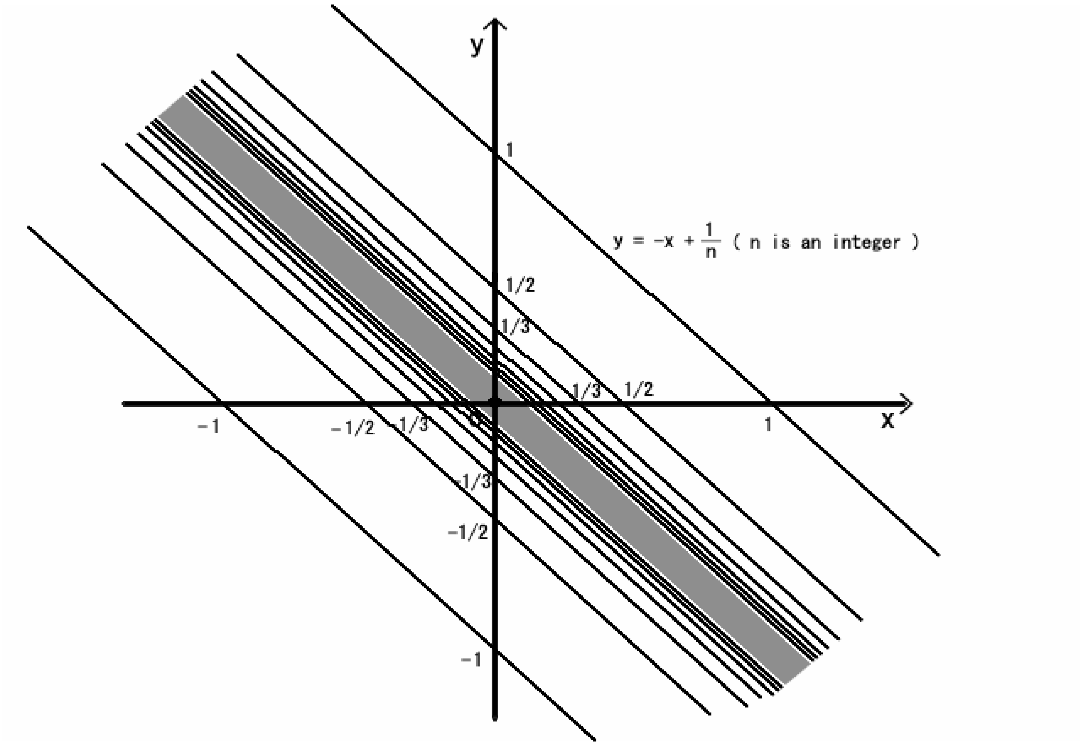
iv) $y \le x^2$



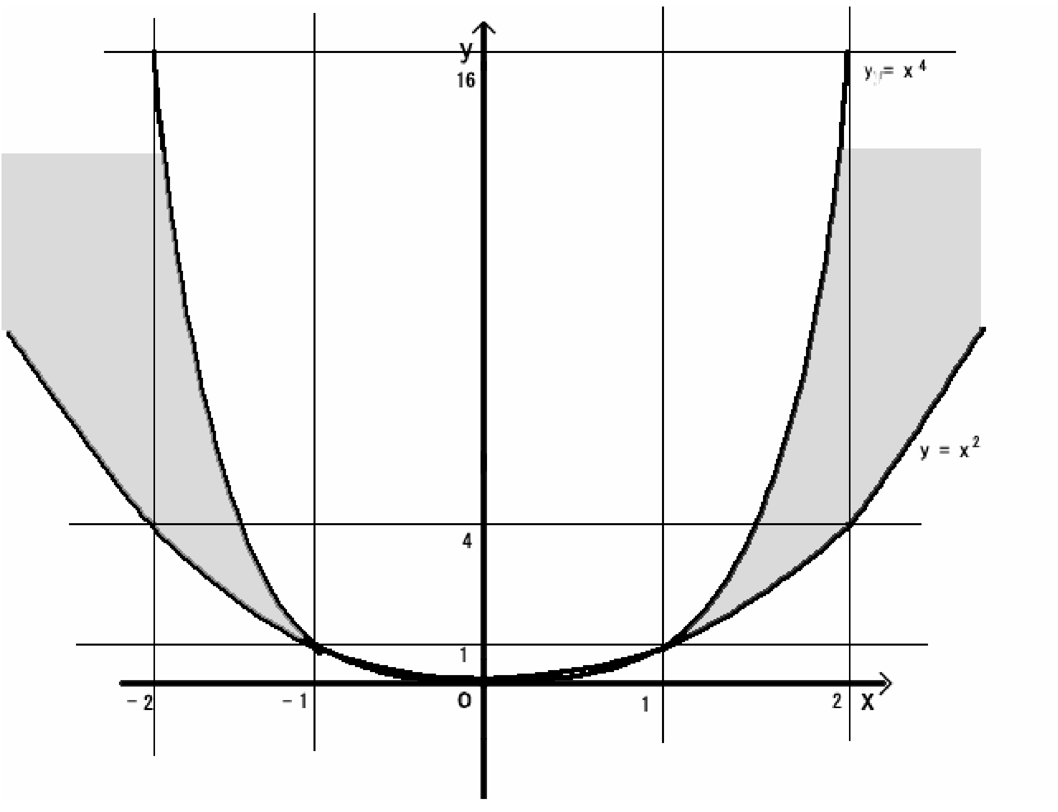
vi) $|x+y|<1$



viii) $\left(\dfrac{1}{x+y}\right)$ is an integer.



x) $x^2 < y < x^4$



Notes: the border is not included, neither the portion between (–1,1) and (1,1).

7. （a）For any numbers A, B and C, with A and B not both 0, show that the set of all (x, y) satisfying $Ax + By + C = 0$ is a straight line (possibly a vertical one).

Proof:

If $B \neq 0$, then $Ax + By + C = 0 \Leftrightarrow y = -\frac{A}{B}x - C$. Obviously, it is a straight line.

If $B = 0$, then $Ax + By + C = 0 \Leftrightarrow x = \frac{C}{A}$, which is also a straight line (vertical).

QED

(b) Show conversely that every straight line, including vertical ones, can be described as the set of all (x, y) satisfying $Ax + By + C = 0$.

Proof:

For any vertical straight line, it must be like $x = x_0 \quad x_0 \in \Re$. Then let

$A = -1, B = 0, C = x_0$, we have $-x + 0 + x_0 = 0 \Leftrightarrow Ax + By + C = 0$

If a line is not straight, it could be written in an equation like

$y = kx + b, \quad where\ k, b \in \Re$, then let $A = k, B = -1, C = b$, obviously,

$kx - y + b = 0 \Leftrightarrow Ax + By + C = 0$

QED

9. (a) Prove, using problem 1-19, that

$$\sqrt{(x_1 + y_1)^2 + (x_2 + y_2)^2} \leq \sqrt{x_1^2 + x_2^2} + \sqrt{y_1^2 + y_2^2}$$

Proof:

$$\sqrt{(x_1 + y_1)^2 + (x_2 + y_2)^2} \leq \sqrt{x_1^2 + x_2^2} + \sqrt{y_1^2 + y_2^2}$$

$$\Leftrightarrow (x_1 + y_1)^2 + (x_2 + y_2)^2 \leq \left(\sqrt{x_1^2 + x_2^2} + \sqrt{y_1^2 + y_2^2}\right)^2 \Leftrightarrow$$

$$x_1^2 + 2x_1y_1 + y_1^2 + x_2^2 + 2x_2y_2 + y_2^2 \leq x_1^2 + x_2^2 + y_1^2 + y_2^2 + 2\sqrt{x_1^2 + x_2^2} \times \sqrt{y_1^2 + y_2^2}$$

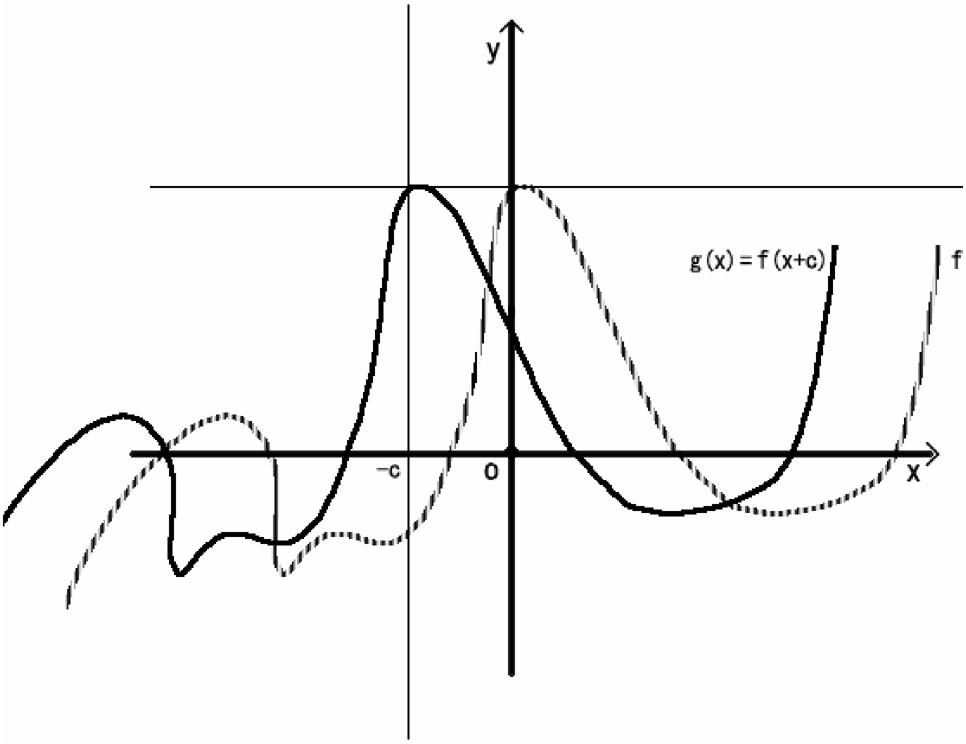
$$\Leftrightarrow 2x_1y_1 + 2x_2y_2 \leq 2\sqrt{x_1^2 + x_2^2} \times \sqrt{y_1^2 + y_2^2}$$

$$\Leftrightarrow x_1y_1 + x_2y_2 \leq \sqrt{x_1^2 + x_2^2} \times \sqrt{y_1^2 + y_2^2}$$

Which was proved previously in problem 1-19. QED.

14. Describe the graph of g in terms of the graph of f if

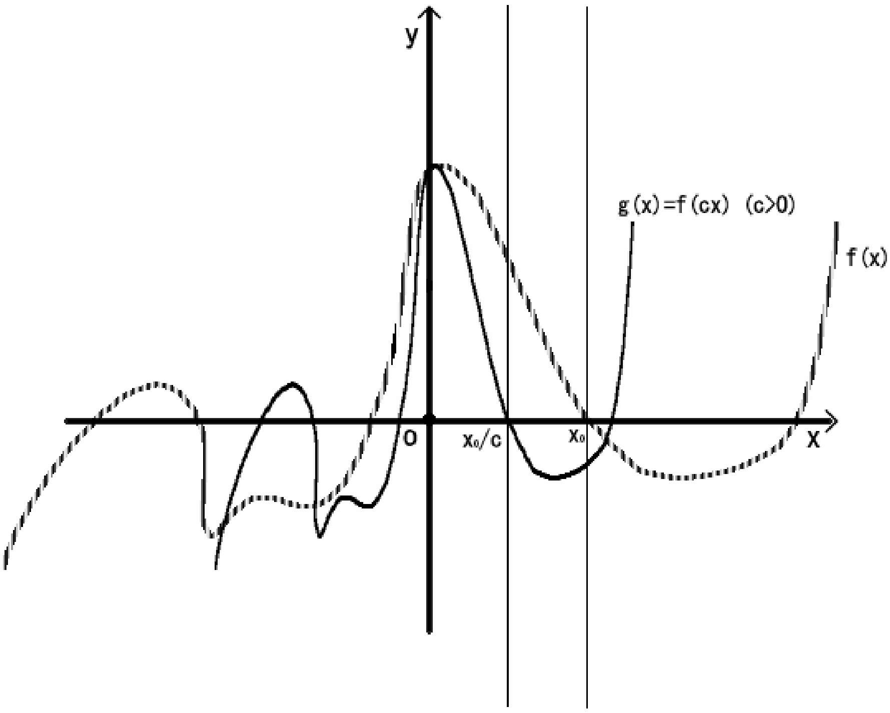
ii) $g(x) = f(x + c)$, which means move graph f(x) c units to the left.



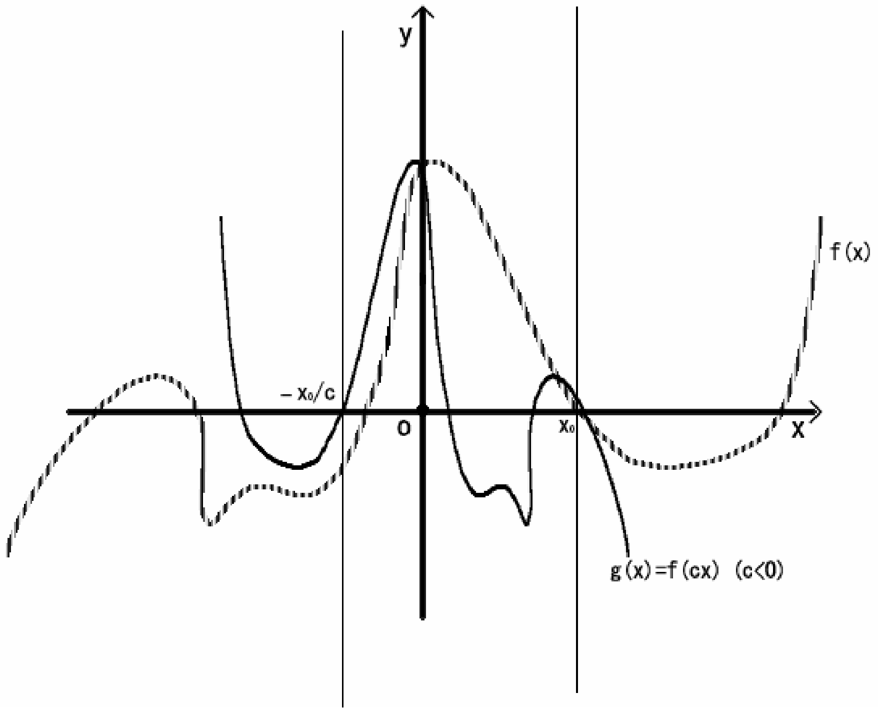
Suppose the dashed line is the graph of f, then the graph of g shows as above. Noticed that

$g(-c) = f(-c+c) = f(0)$

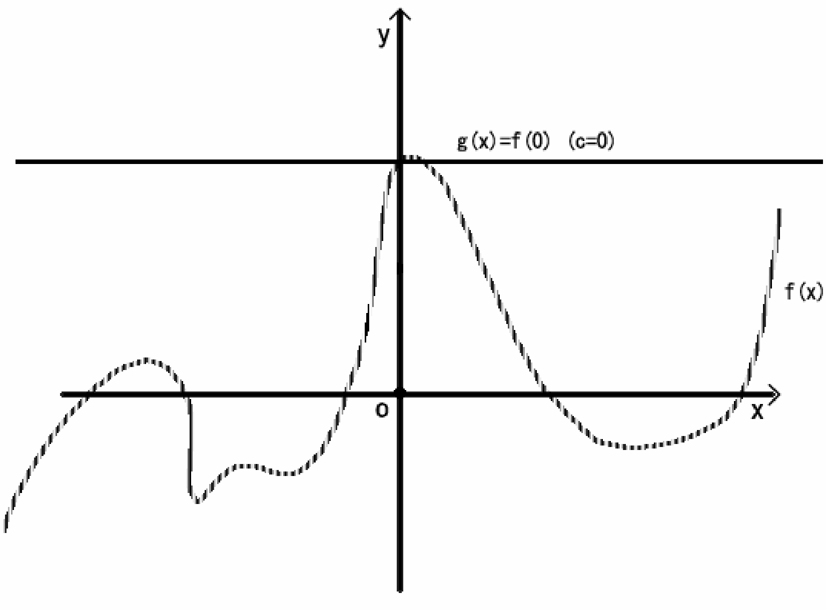
iv) $g(x) = f(cx)$, if c > 0, it means scaling the graph along the x-axis to (1/c) time.



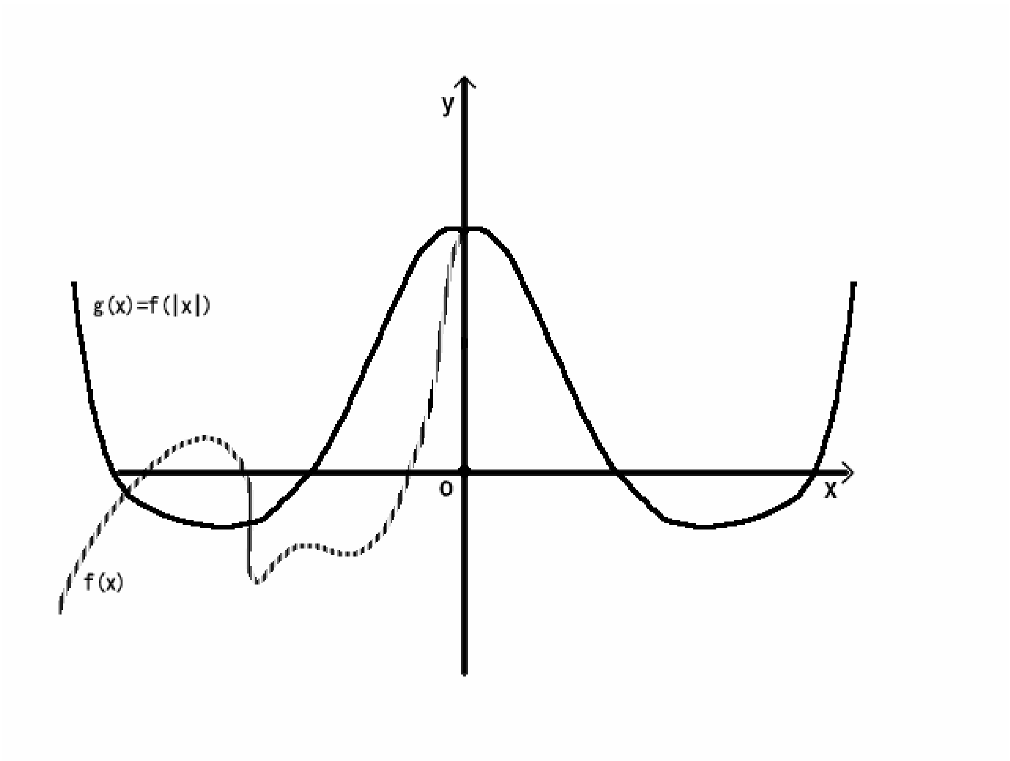
if c<0, it means scaling the graph along the x-axis to (1/c) time and reversing it according to y-axis.



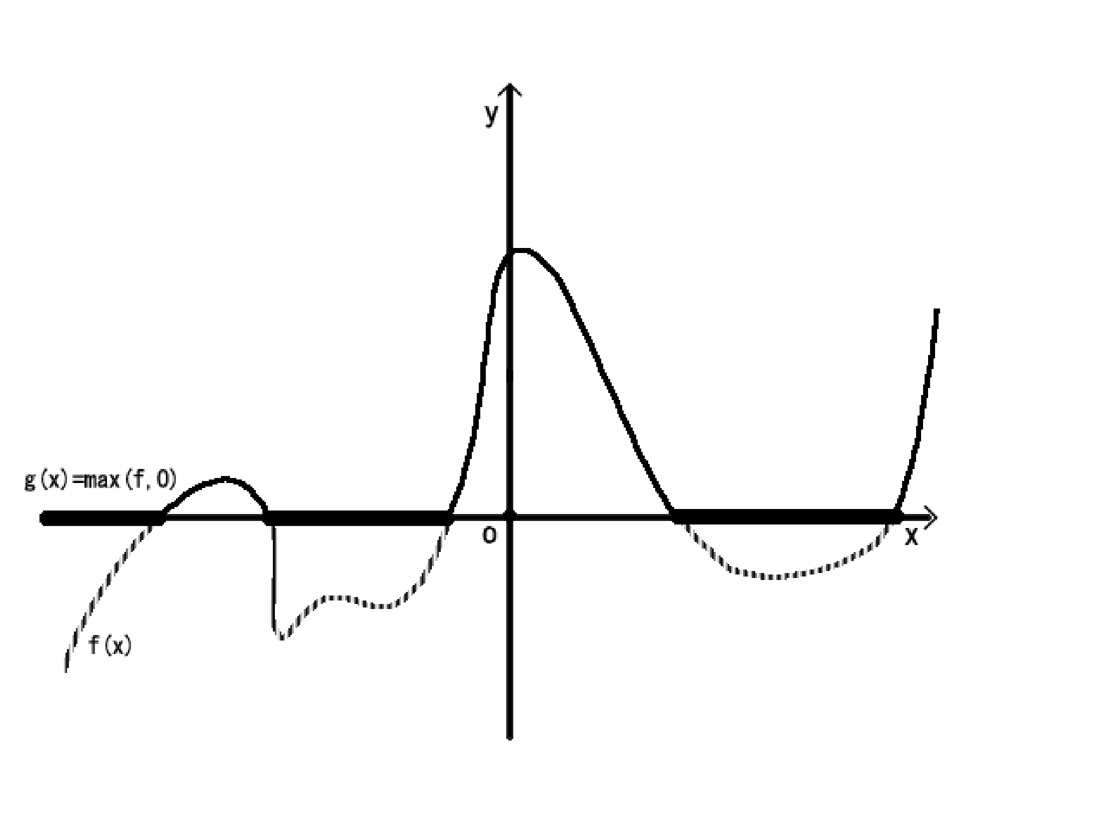
if c = 0, it means a fixed value of f(0).



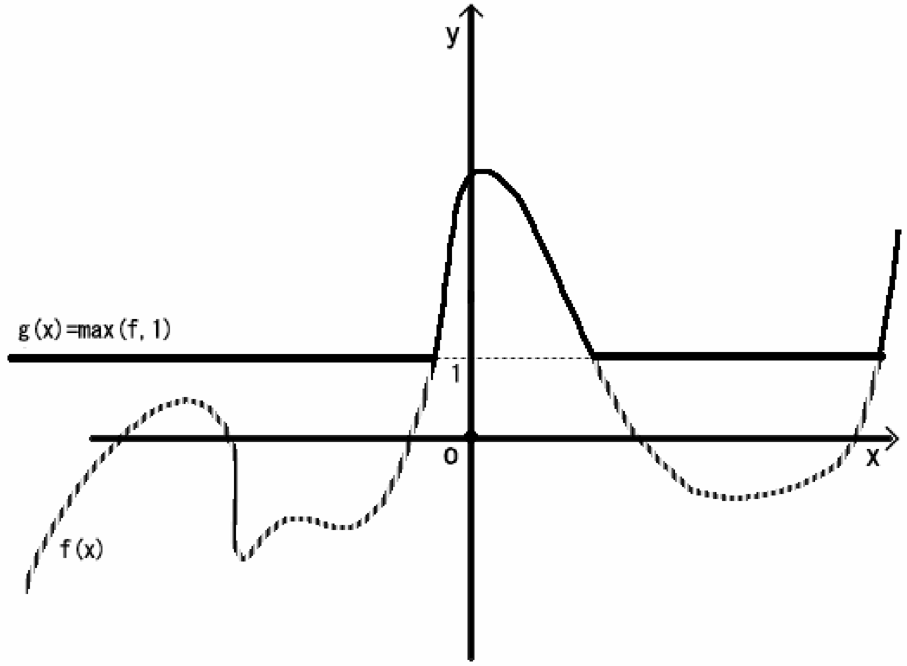
vi) $g(x) = f(|x|)$, it means getting rid of left side of the graph and reversing the right side according to y-axis.



viii) $g(x) = \max(f, 0)$, it means getting rid of the negative y and replacing it with 0.



x) $g(x) = \max(f, 1)$ it means getting rid of any y<1 and replacing it with 1.


y
g(x)=max(f, 1)
1
o
x
f(x)