Chapter 10

2, Solution:

(iii)

$$f'(x)=2\sin\left((x+\sin x)^2\right)\cos\left((x+\sin x)^2\right)\cdot 2(x+\sin x)\cdot(1+\cos x)$$

(vi)

$$f'(x)=31^2(\cos x)^{31^2-1}\cdot(-\sin x)=-31^2\sin x\cdot(\cos x)^{31^2-1}$$

(ix)

$$f'(x)=6\left(x+\sin^5 x\right)^5\cdot\left(1+5\sin^4 x\cdot\cos x\right)$$

(xii)

$$f'(x)=5\left(\left(\left(x^2+x\right)^3+x\right)^4+x\right)^4\cdot\left(4\left(\left(x^2+x\right)^3+x\right)^3\cdot\left(3\left(x^2+x\right)^2\cdot(2x+1)+1\right)+1\right)$$

(xv)

$$f'(x)=\left(\sin x^2\sin^2 x\right)'(1+\sin x)^{-1}+\left((1+\sin x)^{-1}\right)'\left(\sin x^2\sin^2 x\right)$$

$$=\frac{\left(\cos x^2\cdot 2x\cdot\sin^2 x+2\sin x\cdot\cos x\cdot\sin x^2\right)}{1+\sin x}-\frac{\cos x\cdot\sin x^2\cdot\sin^2 x}{(1+\sin x)^2}$$

(xviii)

$$f'(x)=\cos\left(\frac{x}{x-\sin\left(\frac{x}{x-\sin x}\right)}\right)\cdot\left(\frac{x}{x-\sin\left(\frac{x}{x-\sin x}\right)}\right)'$$

$$=\cos\left(\frac{x}{x-\sin\left(\frac{x}{x-\sin x}\right)}\right)\cdot\left(\frac{x\left(x-\sin\left(\frac{x}{x-\sin x}\right)\right)'}{-\left(x-\sin\left(\frac{x}{x-\sin x}\right)\right)^2}+\left(\frac{1}{x-\sin\left(\frac{x}{x-\sin x}\right)}\right)\right)$$

$$=\cos\left(\frac{x}{x-\sin\left(\frac{x}{x-\sin x}\right)}\right)\cdot\left(\frac{x\left(1-\cos\left(\frac{x}{x-\sin x}\right)\cdot\left(\frac{x}{x-\sin x}\right)'\right)}{-\left(x-\sin\left(\frac{x}{x-\sin x}\right)\right)^2}+\left(\frac{1}{x-\sin\left(\frac{x}{x-\sin x}\right)}\right)\right)$$

$$=\cos\left(\frac{x}{x-\sin\left(\frac{x}{x-\sin x}\right)}\right)\cdot\left(\frac{x\left(1-\cos\left(\frac{x}{x-\sin x}\right)\cdot\left(\frac{1}{x-\sin x}+\frac{-x(1-\cos x)}{(x-\sin x)^2}\right)\right)}{-\left(x-\sin\left(\frac{x}{x-\sin x}\right)\right)^2}+\left(\frac{1}{x-\sin\left(\frac{x}{x-\sin x}\right)}\right)\right)$$

6 Solution:

(ii)

$$f'(x) = g'(x \cdot g(a)) \cdot g(a)$$

(iv)

$$f'(x) = g'(x)(x-a) + g(x)$$

(vi)

$$f(x+3) = g(x^2) \Leftrightarrow f(x) = g((x-3)^2) \Leftrightarrow f'(x) = g'\left((x-3)^2\right) \cdot 2(x-3)$$

8 solution:

Since the area of circle is $S = pr^2$, we know: $S_{large} = pr_{large}^{\ 2}$, $S_{small} = pr_{small}^{\ 2}$ and $S_{large} - S_{small} = 9p$.

The circumference of small circle is $C = 2pr_{small} = 2p\sqrt{\frac{S_{small}}{p}} = 2p\sqrt{\frac{S_{large} - 9p}{p}}$.

Thus, $C = 2\sqrt{p}\left(S_{large} - 9p\right)^{\frac{1}{2}}$

Then, $C' = \sqrt{p}\left(S_{large} - 9p\right)^{-\frac{1}{2}} S'_{large}$

We already know $S'_{large} = 10p\ in^2/\sec$, $S_{large} = S_{small} + 9p = 25p\ in^2$

$$C' = \sqrt{p}\left(S_{large} - 9p\right)^{-\frac{1}{2}} S'_{large} = \sqrt{p}\left(25p - 9p\right)^{-\frac{1}{2}} 10p = \frac{5}{2}p\ \ in/\sec$$

9 Solution

The distance between A and B is: $D = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}$.

$$\because y_b = -\sqrt{3x_b}, x_b^{\ 2} + y_b^{\ 2} = 9, y_a = 0$$

$$\therefore D = \sqrt{x_a^{\ 2} - 2x_a x_b + x_b^{\ 2} + y_a^{\ 2} - 2y_a y_b + y_b^{\ 2}}$$

$$= \sqrt{x_a^{\ 2} - 2x_a x_b + \left(x_b^{\ 2} + y_b^{\ 2}\right) + y_a\left(y_a - 2y_b\right)}$$

$$= \sqrt{x_a^{\ 2} - 2x_a x_b + 9 + 0} = \sqrt{x_a^{\ 2} - 2x_a x_b + 9}$$

Look these variable as function of time t, then:

$$D(t) = \sqrt{x_a^{\ 2}(t) - 2x_a(t)x_b(t) + 9}$$

$$D'(t) = \frac{2x_a(t)x_a'(t) - 2\left(x_a'(t)x_b(t) + x_a(t)x_b'(t)\right)}{2\sqrt{x_a^{\ 2}(t) - 2x_a(t)x_b(t) + 9}}$$

$$= \frac{x_a(t)x_a'(t) - x_a'(t)x_b(t) - x_a(t)x_b'(t)}{\sqrt{x_a^2(t) - 2x_a(t)x_b(t) + 9}}$$

On the other hand, we know B is belong to $f(x) = -\sqrt{3x}$ and the distance from the origin is 3, then, at that time:

$$\left.\begin{aligned} y_b &= -\sqrt{3x} \\ x_b^2 + y_b^2 &= 9 \end{aligned}\right\} \Rightarrow B\ is \left(\frac{3}{2}\left(\sqrt{5}-1\right), -\sqrt{\frac{9}{2}\left(\sqrt{5}-1\right)} \right)$$

We know at that time A is at $(5,0)$, also, A moves along the horizontal axis with speed of 3,

$$\Rightarrow x_a'(t) = 3, y_a'(t) = 0$$

So, $D'(t) = \dfrac{x_a(t)x_a'(t) - x_a'(t)x_b(t) - x_a(t)x_b'(t)}{\sqrt{x_a^2(t) - 2x_a(t)x_b(t) + 9}}$

$$= \frac{5 \times 3 - 3 \cdot \frac{3}{2}\left(\sqrt{5}-1\right) - 5x_b'(t)}{\sqrt{49 - 15\sqrt{5}}} = \frac{39 - 9\sqrt{5} - 10x_b'(t)}{2\sqrt{49 - 15\sqrt{5}}}$$

$$\because\ y_b = -\sqrt{3x_b} \Leftrightarrow \frac{dy_b}{dx_b} = -\frac{\sqrt{3}}{2\sqrt{x_b}}$$

$$x_b = \frac{3}{2}\left(\sqrt{5}-1\right) \Rightarrow \frac{dy_b}{dx_b} = -\frac{1}{\sqrt{2\left(\sqrt{5}-1\right)}}$$

$\because$ B moves with a speed 4

$$\therefore \frac{dy_b}{dt} = -\frac{4}{\sqrt{2\sqrt{5}-1}}, \frac{dx_b}{dt} = \frac{4\sqrt{2\sqrt{5}-2}}{\sqrt{2\sqrt{5}-1}}$$

$$\therefore \frac{D(t)}{dt} = \frac{39 - 9\sqrt{5} - 10x_b'(t)}{2\sqrt{49 - 15\sqrt{5}}} = \frac{39 - 9\sqrt{5} - 10\dfrac{4\sqrt{2\sqrt{5}-2}}{\sqrt{2\sqrt{5}-1}}}{2\sqrt{49 - 15\sqrt{5}}}$$

$$\approx 1.8917$$