Dror Bar-Natan: Classes: 2003-04: Math 157 - Analysis I:

Homework Assignment 2

Assigned Tuesday September 16; due Friday September 26, 2PM, at SS 1071

Required reading. All of Spivak Chapter 1.

To be handed in. From Spivak Chapter 1: 11 odd parts, 12 odd parts, 14, and also

1. Show that if $a > 0$, then $ax^2 + bx + c \geq 0$ for all values of x if and only if $b^2 - 4ac \leq 0$.
2. Prove the Cauchy-Schwartz inequality

$$\left(a_1b_1 + a_2b_2 + \cdots a_nb_n\right)^2 \;\leq\; \left(a_1^2 + \cdots + a_n^2\right)\left(b_1^2 + \cdots + b_n^2\right)$$

in two different ways:

(a) Use $2xy \leq x^2 + y^2$ (why is this true?), with

$$x = \frac{|a_i|}{\sqrt{a_1^2 + \cdots + a_n^2}} \qquad y = \frac{|b_i|}{\sqrt{b_1^2 + \cdots + b_n^2}}$$

(b) Consider the expression

$$(a_1x + b_1)^2 + (a_2x + b_2)^2 + \cdots + (a_nx + b_n)^2,$$

collect terms, and apply the result of Problem 1.

Recommended for extra practice. Spivak Chapter 1: 7, 15, 18, 20, 21, 22, 23.

Just for fun. Seen on the web, source unknown (though see http://www.mrc-cbu.cam.ac.uk/˜matt.davis/Cmabrigde/):

> Accdronig to a rscheearch at an Elingsh uinervtisy, it deosn't mttaer in waht oredr the ltteers in a wrod are, the olny iprmoatnt tihng is taht frist and lsat ltteer is at the rghit pclae. The rset can be a toatl mses and you can sitll raed it wouthit porbelm. Tihs is bcuseae we do not raed ervey lteter by it slef but the wrod as a wlohe.