Homework Assignment 3

Assigned Tuesday September 23; due Friday October 3, 2PM, at SS 1071

Required email. The class photo will be on the class' web site in a day or two and you are all required to find it, find yourself in the photo, and send me an email message (either using the feedback form on the class' web site or using my regular email address) with the following information:

- Where are you in the picture? (Use the supplied grid. If your square has more than one face in it, add something like "I'm the guy with the red hair".)
- Your name.
- Your email address.
- Your telephone number (optional).
- Which of the last four pieces of information do you allow me to put on the web? If you don't write anything about this, I'll assume that your location in the photo, your name and your email address are public but that your phone number is to be kept confidential.

Your email is due like the rest of this assignment, on Friday October 3 at 2PM. If you aren't in the picture at all, talk to me after class and I'll take a (small) picture of you on the spot and edit it into the main picture.

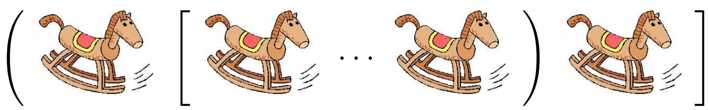
Required reading. All of Spivak Chapters 2 and 3.

To be handed in.
From Spivak Chapter 2: 1, 5.
From Spivak Chapter 3: 6, 13.

Recommended for extra practice.
From Spivak Chapter 2: 3, 4, 12, 22.
From Spivak Chapter 3: 1, 7, 21.

An extra problem: (recommended, but do not submit) Is there a problem with the following inductive proof that all horses are of the same color?

We assert that in all sets with precisely n horses, all horses are of the same color. For $n = 1$, this is obvious: it is clear that in a set with just one horse, all horses are of the same color. Now assume our assertion is true for all sets with $n - 1$ horses, and let us be given a set with n horses in it. By the inductive assumption, the first $n - 1$ of those are of the same color and also the last $n - 1$ of those. Hence they are all of the same color as illustrated below:



(The horses surrounded by round brackets $(\cdots)$ are all of the same color. The horses surrounded by square brackets $[\cdots]$ are all of the same color. Therefore the first and the last horses have the same color as the ones in the middle group, and hence all horses are of the same color.)

Just for fun.

From Spivak Chapter 2: 27, 28.

A little more on Chapter 2, Problem 22:

- We know that if a and b are non-negative then $\sqrt{ab} \leq (a+b)/2$. This is the same as saying that $4ab \leq (a+b)^2$, which is the same as saying that the area of four a by b rectangles is less than or equal to the area of a square with side $a+b$. Can you actually fit four a by b rectangles inside a square of side $a+b$ without overlaps? It's fun and not too hard.

- We know that if a, b and c are non-negative then $\sqrt[3]{abc} \leq (a+b+c)/3$. This is the same as saying that $27abc \leq (a+b+c)^3$, which is the same as saying that the volume of 27 a by b by c rectangular boxes is less than or equal to the volume of a cube with side $a+b+c$. Can you actually fit 27 such a by b by c rectangular boxes inside a cube of side $a+b+c$ without overlaps? This is also fun, but quite hard. You have no chance of doing it without a physical model. Make yourself one!

- The corresponding problem in 4D, involving 256 boxes of size $a \times b \times c \times d$, is actually a little easier, though trickier, than the 3D problem. Can you do it?

- The corresponding problem in 5D, involving 3,125 boxes of size $a \times b \times c \times d \times e$, is an open problem — meaning that nobody knows how to solve it. Can you?

Horse picture from http://lib.allconet.org/story_hour.htm.