Dror Bar-Natan: Classes: 2003-04: Math 157 - Analysis I:

Monsters

Web version: http://www.math.toronto.edu/~drorbn/classes/0304/157AnalysisI/Monsters/Monsters.html

The linear equation and a tame formula:

```
In[1]:= Solve[a x + b == 0, x]
```

Out[1]= $\left\{\left\{x \to -\frac{b}{a}\right\}\right\}$

The quadratic equation and the monster:

```
In[2]:= Solve[a x^2 + b x + c == 0, x] // First
```

Out[2]= $\left\{x \to \frac{-b - \sqrt{b^2 - 4 a c}}{2 a}\right\}$

The cubic equation and the horrible monster:

```
In[3]:= Solve[a x^3 + b x^2 + c x + d == 0, x] // First
```

Out[3]= $$\left\{x \to -\frac{b}{3 a} - \frac{2^{1/3} (-b^2 + 3 a c)}{3 a \left(-2 b^3 + 9 a b c - 27 a^2 d + \sqrt{4 (-b^2 + 3 a c)^3 + (-2 b^3 + 9 a b c - 27 a^2 d)^2}\right)^{1/3}} + \frac{\left(-2 b^3 + 9 a b c - 27 a^2 d + \sqrt{4 (-b^2 + 3 a c)^3 + (-2 b^3 + 9 a b c - 27 a^2 d)^2}\right)^{1/3}}{3\, 2^{1/3} a}\right\}$$

The quartic equation and the truly horrible monster:

In[4]:= **Solve[a x^4 + b x^3 + c x^2 + d x + e == 0, x] // First**

Out[4]=
$$\Bigg\{x \to -\frac{b}{4\,a} - \frac{1}{2}\sqrt{\frac{b^2}{4\,a^2} - \frac{2\,c}{3\,a} + \left(2^{1/3}\,(c^2 - 3\,b\,d + 12\,a\,e)\right) \Big/ \left(3\,a\left(2\,c^3 - 9\,b\,c\,d + 27\,a\,d^2 + 27\,b^2\,e - 72\,a\,c\,e + \sqrt{-4\,(c^2 - 3\,b\,d + 12\,a\,e)^3 + (2\,c^3 - 9\,b\,c\,d + 27\,a\,d^2 + 27\,b^2\,e - 72\,a\,c\,e)^2}\right)^{1/3}\right) + \frac{1}{3\,2^{1/3}\,a}\left(\left(2\,c^3 - 9\,b\,c\,d + 27\,a\,d^2 + 27\,b^2\,e - 72\,a\,c\,e + \sqrt{-4\,(c^2 - 3\,b\,d + 12\,a\,e)^3 + (2\,c^3 - 9\,b\,c\,d + 27\,a\,d^2 + 27\,b^2\,e - 72\,a\,c\,e)^2}\right)^{1/3}\right)} - \frac{1}{2}\sqrt{\frac{b^2}{2\,a^2} - \frac{4\,c}{3\,a} - \left(2^{1/3}\,(c^2 - 3\,b\,d + 12\,a\,e)\right) \Big/ \left(3\,a\left(2\,c^3 - 9\,b\,c\,d + 27\,a\,d^2 + 27\,b^2\,e - 72\,a\,c\,e + \sqrt{-4\,(c^2 - 3\,b\,d + 12\,a\,e)^3 + (2\,c^3 - 9\,b\,c\,d + 27\,a\,d^2 + 27\,b^2\,e - 72\,a\,c\,e)^2}\right)^{1/3}\right) - \frac{1}{3\,2^{1/3}\,a}\left(\left(2\,c^3 - 9\,b\,c\,d + 27\,a\,d^2 + 27\,b^2\,e - 72\,a\,c\,e + \sqrt{-4\,(c^2 - 3\,b\,d + 12\,a\,e)^3 + (2\,c^3 - 9\,b\,c\,d + 27\,a\,d^2 + 27\,b^2\,e - 72\,a\,c\,e)^2}\right)^{1/3}\right) - \left(-\frac{b^3}{a^3} + \frac{4\,b\,c}{a^2} - \frac{8\,d}{a}\right) \Big/ \left(4\sqrt{\frac{b^2}{4\,a^2} - \frac{2\,c}{3\,a} + \left(2^{1/3}\,(c^2 - 3\,b\,d + 12\,a\,e)\right) \Big/ \left(3\,a\left(2\,c^3 - 9\,b\,c\,d + 27\,a\,d^2 + 27\,b^2\,e - 72\,a\,c\,e + \sqrt{-4\,(c^2 - 3\,b\,d + 12\,a\,e)^3 + (2\,c^3 - 9\,b\,c\,d + 27\,a\,d^2 + 27\,b^2\,e - 72\,a\,c\,e)^2}\right)^{1/3}\right) + \frac{1}{3\,2^{1/3}\,a}\left(\left(2\,c^3 - 9\,b\,c\,d + 27\,a\,d^2 + 27\,b^2\,e - 72\,a\,c\,e + \sqrt{-4\,(c^2 - 3\,b\,d + 12\,a\,e)^3 + (2\,c^3 - 9\,b\,c\,d + 27\,a\,d^2 + 27\,b^2\,e - 72\,a\,c\,e)^2}\right)^{1/3}\right)}\right)}\Bigg\}$$

The quintic equation; we're dead now:

In[5]:= **Solve[a x^5 + b x^4 + c x^3 + d x^2 + e x + f == 0, x] // First**

Out[5]= {x → Root[f + e #1 + d #1^2 + c #1^3 + b #1^4 + a #1^5 &, 1]}