Homework Assignment 5

Assigned Thursday October 16; due Thursday October 23 in class.

Required reading. Sections 1 and 2 of my paper On the Vassiliev Knot Invariants.

To be handed in.

1. Let Δ be the "doubling" (also called "cabling") operation on knots, which takes a framed knot and replaces it by a 2-component link by "replacing every line by a double line" in an obvious manner.
   (a) Show that if V is a type m invariant of 2-component links then $V \circ \Delta$ is a type m invariant of knots.
   (b) Find a map $\Delta : \mathcal{A}(\bigcirc) \to \mathcal{A}(\bigcirc\bigcirc)$ (sorry for the "operator overloading") for which $W_{V\circ\Delta} = W_V \circ \Delta$ for all such m and V. (Verify that you proposed map respects the $4T$ relation!)
2. If D is a chord diagram, let $X(D)$ be the number of "chord crossings" in D (so for example, $X(\otimes) = 1$).
   (a) Does $X : \mathcal{D} \to \mathbb{Z}$ satisfy the $4T$ relation?
   (b) Let m by a natural number. Can you find a type m knot invariant V for which $W_V = X$?

Idea for a good deed. Tell us about the Milnor-Moore theorem: *A connected commutative and co-commutative graded Hopf algebra over a field of characteristic* 0 *which is of finite type, is the symmetric algebra over the vector space of its primitives.*