Dror Bar-Natan: Classes: 2004-05: Math 157 - Analysis I:

Homework Assignment 22

Assigned Tuesday March 22; due Friday April 1, 2PM, at SS 1071

Required reading. All of Spivak's Chapters 22 and 23.

To be handed in. From Spivak Chapter 23: Problems 1 (parts divisible by 4), 12, 23.

Recommended for extra practice. From Spivak Chapter 23: Problems 1 (the rest), 5, 20, 21.

In class review problem(s) (to be solved in class on Thursday March 31):

- Prove that the following sums diverge: (Hint: Use problem 20.)

$$\sum_{n=1}^{\infty}\frac{1}{n};\qquad \sum_{n=2}^{\infty}\frac{1}{n(\log n)};\qquad \sum_{n=3}^{\infty}\frac{1}{n(\log n)(\log\log n)};$$

$$\sum_{n=16}^{\infty}\frac{1}{n(\log n)(\log\log n)(\log\log\log n)};\quad \dots$$

- Prove that the following sums converge: (Hint: Use problem 20.)

$$\sum_{n=1}^{\infty}\frac{1}{n^{1.01}};\qquad \sum_{n=2}^{\infty}\frac{1}{n(\log n)^{1.01}};\qquad \sum_{n=3}^{\infty}\frac{1}{n(\log n)(\log\log n)^{1.01}};$$

$$\sum_{n=16}^{\infty}\frac{1}{n(\log n)(\log\log n)(\log\log\log n)^{1.01}};\quad \dots$$

Just for fun. In this question we always assume that $a_n > 0$ and $b_n > 0$. Let's say that a sequence a_n is "much bigger" than a sequence b_n if $\lim_{n\to\infty} a_n/b_n = \infty$. Likewise let's say that a sequence a_n is "much smaller" than a sequence b_n if $\lim_{n\to\infty} a_n/b_n = 0$. Prove that for every convergent series $\sum b_n$ there is a much bigger sequence a_n for which $\sum a_n$ is also convergent, and that for every divergent series $\sum b_n$ there is a much smaller sequence a_n for which $\sum a_n$ is also divergent. (Thus you can forever search in vain for that fine line between good and evil; it just isn't there).

Advertisement 1'. A short addendum to Advertisement 1 of HW21:

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Date: Sun, 20 Mar 2005 21:53:48 -0500

Dr. Bar-Natan:

Thank you for posting our announcement on your website, the advertising
is greatly appreciated! However, a minor note: technically, this event
*does* include free food - 5 meals (not to mention a T-shirt!) are
included in the $60 registration fee, truly a fantastic bargain!;)

Cheers,
Erica Blom
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