Homework Assignment 23

Assigned Tuesday March 29; due Friday April 8, 2PM, at SS 1071

Required reading. All of Spivak's Chapters 23 and 24.

To be handed in. From Spivak Chapter 24: Problems 2 (odd parts), 5 (odd parts), 17, 23.

Recommended for extra practice. From Spivak Chapter 24: 2 (even parts), 5 (even parts), 12, 15, 22, 24.

Just for fun 1. The series

$$\sum_{n=1}^{\infty} \frac{1}{2^n} \sin 3^n x$$

is quite bizarre, as it converges uniformly to a continuous function $f(x)$, yet that function f is so bumpy that it is not differentiable for *any* x.

- Use theorems from class to show that f is indeed continuous and that the converegence of the series is indeed uniform.
- Try to differentiate the series term by term and convince yourself that after differentiation, there is no reason to expect the resulting series to be convergent.
- Check numerically that f is not differentiable for any x by computing $(f(x+h)-f(x))/h$ on your computer or calculator for very small values of h and for a number of different choices for x.
- Plot $y = f(x)$ well enough to see that it is indeed very bumpy.

Just for fun 2. Another fun example for the use of uniform convergence is the construction of a space-filling curve — a continuous function f whose domain is the unit interval I and whose range is the *entire* unit square $I \times I$. (On first sight — does this seem possible??)

- This would be a function whose input is a single number and whose output is a pair of numbers. Convince yourself that the words "continuity", "convergence" and "uniform convergence" can be given a meaning in this context, and that they have similar properties as in the case of ordinary functions.
- Do a web search to find (many!) pictures of space-filling curves (aka "Peano curves").