

Last Handout

web version:

<http://www.math.toronto.edu/~drorbn/classes/0405/157AnalysisI/LastHandout/LastHandout.html>

Things we didn't reach.

1. **Euler's Formula** (aka "the most beautiful formula in mathematics"):

$$e^{i\pi} + 1 = 0,$$

where π is the ratio of the circumference of a circle to its diameter, e is the basis of the natural logarithms, i is the square root of -1 , and 0 and 1 are, well, you know. Less beautiful but more significant (and equally weird) is de Moivre's formula

$$e^{ix} = \cos x + i \sin x.$$

2. **Question.** Find a formula for the n 'th term F_n in the sequence 1, 1, 2, 3, 5, 8, 13, 21, 34, etc.

Hint. It helps to know that

$$\sum_{n=1}^{\infty} F_n x^n = \frac{x}{1 - x - x^2}.$$

3. **Bessel's function** J_0 is the solution of the differential equation $x^2 J_0'' + x J_0' + x^2 J_0 = 0$ with $J_0(0) = 1$ and $J_0'(0) = 0$. A formula for J_0 is

$$J_0(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2^k k!)^2}.$$

A few things we really have to say.

Definition. We say that a sequence of functions f_n converges to a function f *uniformly* on an interval I if for every $\epsilon > 0$ there is some N so that whenever $n > N$ and $x \in I$, we have that $|f_n(x) - f(x)| < \epsilon$. Likewise we define *uniform convergence* for series $\sum_{n=1}^{\infty} f_n(x)$.

Theorem 1. If $f = \sum f_n$ uniformly on I and if for every n the function f_n is continuous on I , then f is also continuous on I .

Theorem 2. If $f = \sum f_n$ uniformly on I and if for every n the function f_n is integrable on I then f is integrable on I and $\int_I f = \sum \int_I f_n$.

Theorem 3. A similar though slightly more complicated statement holds for derivatives.

Theorem 4. If the series $\sum a_n x^n$ is convergent for some $x = x_0$, then it is uniformly and absolutely convergent on $[-(x_0 - \epsilon), x_0 - \epsilon]$, for every $\epsilon > 0$. Thus "all the good things" happen for functions such as $J_0(x)$.

On the Final Exam. It will take place, as dictated by the Higher Authorities, on Wednesday May 4, 7–10PM (late!) at the Upper Small Gymnasium, Benson Building, 320 Huron Street (across from Sidney Smith, south of Harbord Street), Third Floor. The material is very easy to define: *Everything*. In more details, this is chapters 1–15, 18–20 and 22–24 of Spivak’s book, minus appendices plus the appendix to chapter 19 plus some extra material on convexity (as it was discussed in class). If any question will relate to chapter 24, it will be relatively simple and will not require knowing proofs.

Preparing for the Final Exam.

- Re-read your notes and make sure that you understand *everything*.
- Re-read the relevant chapters of Spivak’s book and make sure that you understand *everything*.
- You may want to prepare a list of all topics touched in class (you may reach 200 or even 400), and you may want to go over this list several times until you are sure you understand everything in full.
- Make sure that you can solve every homework problem assigned or recommended.
- Take a good look at exams, sample exams and exam solutions from previous years. (Scroll down to the bottom of this class’ web site and find the relevant links).
- Come to my office hours on Monday and Tuesday May 2nd and 3rd, from 10AM until 1PM (or even later, if there’s demand), at the Math Aid Centre, SS 1071.
- It is much more fun to work in a group!

An often-asked question is “Do we need to know proofs?”. The answer is **Absolutely**. Proofs are often the deepest form of understanding, and hence they are largely what this class is about. The ones I show in class are precisely those that I think are the most important ones, thus they are the ones you **definitely** need to know.

Good Luck!!

Last Comment. Please remember that absolutely all homework is due Friday April 15, 2PM, at the Math Aid Centre.