Math 157 Analysis I — Term Exam 1

University of Toronto, October 18, 2004

Name: _____ Student ID: _____

Solve 4 of the following 5 problems. Each problem is worth 25 points. If you solve more than 4 problems indicate very clearly which ones you want graded; otherwise a random one will left out at grading and it may be your best one! Write your answers in the space below the problems and on the front sides of the extra pages; use the back of the pages for scratch paper. Only work appearing on the front side of pages will be graded. Write your name and student number on each page. If you need more paper please ask the tutors. You have an hour and 50 minutes.

Allowed Material: Any calculating device that is not capable of displaying text.

Good Luck!

For Grading Use Only

1	/25	4	/25
2	/25	5	/25
3	/25	Total	/100

Web version: http://www.math.toronto.edu/~drorbn/classes/0405/157AnalysisI/TE1/Exam.html

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Consent Form. (Don't sign if you are uncomfortable) I have an account at CCNET, I have set a password on that account and I hereby agree that my exam grades for Math 157 (UofT, 2004-5) will be posted on CCNET. The grades will be viewable only by the CCNET staff, the course staff and via my account. I understand that the security of this arrangement cannot be guaranteed.

Date Name Student Number Signature

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Problem 1. Find formulas for $\sin\alpha$, $\cos\alpha$ and $\tan\alpha$ in terms of $\tan\frac{\alpha}{2}$. (You may use any formula proven in class; you need to quote such formulae, though you don't need to reprove them).

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Problem 2.

1. Let k be a natural number. Prove that any natural number n can be written in a unique way in the form $n = qk + r$, where q and r are integers and $0 \leq r < k$.

2. We say that a natural number n is "divisible by 3" if $n/3$ is again a natural number. Prove that n is divisible by 3 if and only if n^2 is divisible by 3.

3. We say that a natural number n is "divisible by 4" if $n/4$ is again a natural number. Is it true that n is divisible by 4 if and only if n^2 is divisible by 4?

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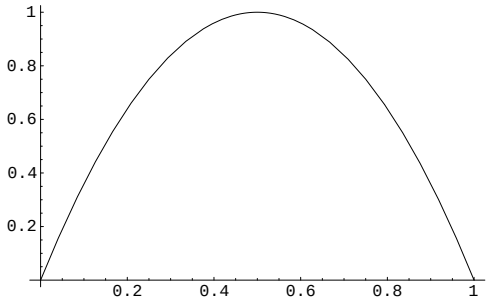

Problem 3. A function $f(x)$ is defined for $0 \leq x \leq 1$ and has the graph plotted above.

1. What are $f(0)$, $f(0.5)$ and $f(1)$?
2. Let g be the function $f \circ f$. What are $g(0)$, $g(0.5)$ and $g(1)$?
3. Are there any values of x for which $g(x) = 1$? How many such x's are there? Roughly what are they?
4. Plot the graph of the function g. (The general shape of your plot should be clear and correct, though numerical details need not be precise).
5. (5 points bonus, will be given only to excellent solutions and may raise your overall exam grade to 105!) Plot the graphs of the functions $g \circ f$ and $g \circ g$.

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Problem 4.

1. Define "$\lim_{x\to a} f(x) = l$" and "$\lim_{x\to a+} f(x) = l$".
2. Prove that if $\lim_{x\to a+} f(x) = l$ and $\lim_{x\to a-} f(x) = l$ then $\lim_{x\to a} f(x) = l$.
3. Prove that if $\lim_{x\to a} f(x) = l$ then $\lim_{x\to a+} f(x) = l$ and $\lim_{x\to a-} f(x) = l$.
4. Draw the graph of some function for which $\lim_{x\to a+} f(x) = 0$ and $\lim_{x\to a-} f(x) = 1$.

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Problem 5. Give examples to show that the following definitions of $\lim_{x \to a} f(x) = l$ do not agree with the standard one:

1. For all $\delta > 0$ there is an $\epsilon > 0$ such that if $0 < |x - a| < \delta$, then $|f(x) - l| < \epsilon$.
2. For all $\epsilon > 0$ there is a $\delta > 0$ such that if $|f(x) - l| < \epsilon$, then $0 < |x - a| < \delta$.

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