Math 157 Analysis I — Term Exam 2

University of Toronto, November 29, 2004

Name: ______________________________ Student ID: __________________

Solve all of the following 5 problems. Each problem is worth 20 points. Write your answers in the space below the problems and on the front sides of the extra pages; use the back of the pages for scratch paper. Only work appearing on the front side of pages will be graded. Write your name and student number on each page. If you need more paper please ask the tutors. You have an hour and 50 minutes.

Allowed Material: Any calculating device that is not capable of displaying text.

Good Luck!

For Grading Use Only

1	/20	4	/20
2	/20	5	/20
3	/20	Total	/100

Web version: http://www.math.toronto.edu/~drorbn/classes/0405/157AnalysisI/TE2/Exam.html

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Second Chance Consent Form. (For use only if you didn't sign this consent form before; don't sign if you are uncomfortable) I have an account at CCNET, I have set a password on that account and I hereby agree that my exam grades for Math 157 (UofT, 2004-5) will be posted on CCNET. The grades will be viewable only by the CCNET staff, the course staff and via my account. I understand that the security of this arrangement cannot be guaranteed.

Date Name Student Number Signature

Name: ________________________________ Student ID: ___________________

Problem 1. Let f and g be continuous functions defined on all of $\mathbb{R}$.

1. Prove that if $f(a) \neq g(a)$ for some $a \in \mathbb{R}$, then there is a number $\delta > 0$ such that $f(x) \neq g(x)$ whenever $|x - a| < \delta$.

2. Prove that if two continuous functions are equal over the rationals then they are always equal. That is, if $f(r) = g(r)$ for every $r \in \mathbb{Q}$ then $f(x) = g(x)$ for all $x \in \mathbb{R}$.

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Problem 2. Let f be a continuous function defined on all of $\mathbb{R}$, and assume that $f(x)$ is rational for every $x \in \mathbb{R}$. Prove that f is a constant function.

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Problem 3. We say that a function f is *locally bounded* on some interval I if for every $x \in I$ there is an $\epsilon > 0$ so that f is bounded on $I \cap (x - \epsilon, x + \epsilon)$. Let f be a locally bounded function on the interval $[a, b]$ and let $A = \{x \in [a, b] : f \text{ is bounded on } [a, x]\}$ and $c = \sup A$.

1. Justify the definition of c: How do we know that $\sup A$ exists?

2. Prove that $c > a$.

3. Prove that $c = b$.

4. Prove that $b \in A$.

5. Can you summarize these results with one catchy phrase?

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Problem 4.

1. Define "f is differentiable at a".
2. Prove that if f is differentiable at a then it is also continuous at a.

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Problem 5. Draw an approximate graph of the function $f(x) = \dfrac{x^2}{x^2 - 1}$ making sure to clearly indicate (along with clear justifications) the domain of definition of f, its x-intercepts and its y-intercepts (if any), the behaviour of f at $\pm\infty$ and near points at which f is undefined (if any), intervals on which f is increasing/decreasing, its local minima/maxima (if any) and intervals on which f is convex/concave.