# Math 157 Analysis I — Term Exam 3

University of Toronto, February 7, 2005

Name: ______________________________ Student ID: __________________

**Solve all of the following 4 problems.** Each problem is worth 25 points. Write your answers in the space below the problems and on the front sides of the extra pages; use the back of the pages for scratch paper. Only work appearing on the front side of pages will be graded. Write your name and student number on each page. If you need more paper please ask the tutors. You have an hour and 50 minutes.

**Allowed Material:** Any calculating device that is not capable of displaying text.

**Good Luck!**

For Grading Use Only

| | |
|---|---|
| 1 | /25 |
| 2 | /25 |
| 3 | /25 |
| 4 | /25 |
| Total | /100 |

Web version: http://www.math.toronto.edu/~drorbn/classes/0405/157AnalysisI/TE3/Exam.html

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**Problem 1.**

1. Compute $\int_0^1 \sqrt{x}\, dx$.

2. Compute $\int_0^\pi \sin x\, dx$.

3. For $x \geq 0$, compute $\frac{d}{dx} \int_{x^3}^{157} \sqrt{t}\, dt$.

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**Problem 2.**

1. Perhaps using L'Hôpital's law, compute $\lim_{x\to 0} \frac{\sin x}{x}$ and $\lim_{x\to 0} \frac{1-\cos x}{x^2}$.
2. Use these results to give educated guesses for the values of $\sin 0.1$ and $\cos 0.1$ (no calculators, please).

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**Problem 3.**

1. State the "one partition for every $\epsilon$" criterion of the integrability of a bounded function $f$ defined on an interval $[a, b]$.

2. Let $f$ be an increasing function on $[0, 1]$ and let $P_n$ be the partition defined by $t_i = i/n$, for $i = 0, 1, \ldots, n$. Write simple formulas for $U(f, P_n)$ and for $L(f, P_n)$.

3. Under the same conditions, write a very simple formula for $U(f, P_n) - L(f, P_n)$.

4. Prove that an increasing function on $[0, 1]$ is integrable.

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**Problem 4.**

1. Show that the function $f(x) = 3x - x^3$ is monotone on the interval $[-1, 1]$.
2. Deduce that for every $c \in [-2, 2]$ the equation $3x - x^3 = c$ has a unique solution $x$ in the range $-1 \leq x \leq 1$.
3. For $c \in [-2, 2]$, let $g(c)$ be the unique $x$ in the range $-1 \leq x \leq 1$ for which $3x - x^3 = c$. Write a formula for $g'(c)$ and simplify it as much as you can. Your end result may still contain $g(c)$ in it, but not $f$, $f'$ or $g'$.