Homework Assignment 11

Assigned Thursday March 24; due Thursday April 7, 3PM, in class

Required reading. Read, reread and rereread your notes to this point, and make sure that you really, really really, really really really understand everything in them. Do the same every week! Also, read Hatcher's pages 166–176 and 185–217.

Solve the following problems. (But submit only the underlined ones). In Hatcher's book, problems 1, <u>2</u>, 3, 4, <u>5</u>, 10, <u>11</u> on pages 176–177 (S^∞ and $\mathbb{RP}^\infty$ are defined on pages 6–7).

Just for fun. Let γ_0 be a nicely embedded circle in $\mathbb{R}^3$ ("nicely" means smoothly or even piecewise polygonally with finitely many pieces, if you wish). We know that $H_1(\mathbb{R}^3-\gamma_0)=\mathbb{Z}$.

- Understand this fact straight from the definition of homology on a pictorial level, without referring to any theorems whose proofs you cannot hold in your head in one piece.
- Now let γ_1 be another nicely embedded circle in $\mathbb{R}^3$, disjoint from γ_0 (so the two circles together form a 2-component link). Then γ_1 represents a class in $H_1(\mathbb{R}^3-\gamma_0)$ and hence an integer called $\ell(\gamma_0,\gamma_1)$. Use your understanding from the previous part to give a simple method to compute $\ell(\gamma_0,\gamma_1)$ on a pictorial level.
- Prove that $\ell(\gamma_0,\gamma_1)=\ell(\gamma_1,\gamma_0)$.
- How is $\ell(\gamma_0,\gamma_1)$ related to the "linking number" defined in class a while ago, using the degree of a map $T^2\to S^2$?