# MAT 247
# ASSIGNMENT 4
# DUE THURSDAY FEBRUARY 10

(1) (Axler 7.9) Prove that a normal operator on a complex inner product space is self-adjoint iff all of its eigenvalues are real.
(2) (Axler 7.16) Give an example of an operator $T$ on a inner product space $V$ with a subspace $W$ such that $W$ is $T$-invariant, but $W^{\perp}$ is not $T$-invariant.
(3) Let $V$ be an inner product space and let $W$ be a subspace. We have $V = W \oplus W^{\perp}$. Define a linear operator $T : V \to V$ by $T(w + u) = w - u$ if $w \in W$ and $u \in W^{\perp}$. Prove that $T$ is an isometry and is self-adjoint.
(4) Prove the converse to (3). More precisely, suppose that $V$ is an inner product space and $T : V \to V$ is a self-adjoint isometry. Show that there exists a subspace $W$ of $V$ such that $T(w+u) = w - u$, whenever $w \in W$ and $u \in W^{\perp}$.