MAT 247, Winter 2014
Assignment 1
Due Jan 14

January 6, 2014

1. Suppose that A, B are $n \times n$ matrices. Assume that there exists an invertible $n \times n$ matrix Q such that $Q^{-1}AQ = B$. Prove that there exists a vector space V and a linear operator $T : V \to V$ such that A and B are both matrices for T (with respect to two different bases).

2. Consider the matrix
$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$
   (a) Consider A as a linear operator on $\mathbb{R}^2$ and find a basis for $\mathbb{R}^2$ consisting of eigenvectors for this linear operators.
   (b) Find an invertible matrix Q such that $Q^{-1}AQ$ is diagonal.

3. Prove that the following conditions on a square matrix A are equivalent.
   (a) A is a scalar multiple of the identity matrix.
   (b) Every vector is an eigenvector for A.
   (c) A is diagonalizable and has only one eigenvalue.
   (d) There are no matrices (other than A) which are similar to A.

4. For each of the following complex matrices A, determine if there exists a complex matrix B such that $B^2 = A$. (Hint: use Jordan form.)

(a)

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

(b)

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

5. Recall that an $n \times n$ complex matrix A is called nilpotent if 0 is its only eigenvalue. How many 5×5 nilpotent Jordan form matrices are there?