

Midterm review

MAT 247

The midterm will be held on Tuesday March 4, in EX 300 (that's the exam centre), 1:15 pm - 3 pm.

The midterm will cover all material presented in class before reading week, as well as Assignments 1-6.

Study the following definitions: generalized eigenvector, generalized eigenspace, minimal polynomial, Jordan form matrix, nilpotent operator, determinant, permutation, characteristic polynomial, T -cyclic vector space, dual vector space, quotient space, bilinear pairing, non-degenerate bilinear pairing.

Study all theorems presented in class, but you only need to study the proofs of the following results. Let $T : V \rightarrow V$ be a linear operator.

1. The roots of the minimal polynomial of T are the eigenvalues of T .
2. T is diagonalizable if and only if the minimal polynomial of T splits into distinct linear factors.
3. If $\lambda_1 \neq \lambda_2$, then $K_{\lambda_1} \cap K_{\lambda_2} = 0$ (K_{λ} denotes the generalized eigenspace).
4. If T is nilpotent, then it has a Jordan basis.
5. If two Jordan form matrices are similar, then they have the same set of blocks.
6. $\det T \neq 0$ if and only if T is invertible.
7. The Cayley-Hamilton theorem ($q(T) = 0$, where $q(t)$ is the characteristic polynomial).
8. V is T -cyclic if and only if the Jordan form matrix of T has one block for each eigenvalue.