MAT347Y1 HW3 Marking Scheme

Friday, October 9

Total: 30 points.

1.7.19: 4 points.

- (1) Injectivity
- (1) Surjectivity
- (2) Lagrange's Theorem

2.1.4: 3 points.

2.1.9: 4 points.

- (1) Nonempty subset (suffices to show it contains the identity)
- (3) Closed under xy^{-1} (or under multiplication and inverses separately)

2.3.26:

(a) 3 points.

- (1) σ_a is a homomorphism
- (1) (a, n) = 1 implies bijective
- (1) Bijective implies (a, n) = 1
- (b) 2 points. (one for each direction)
- (c) 2 points.
- (d) 3 points.
 - (1) $\overline{a} \mapsto \sigma_a$ is well-defined
 - (1) $\sigma_a \circ \sigma_b = \sigma_{ab}$ (so $\overline{a} \mapsto \sigma_a$ is a homomorphism)
 - (1) $\overline{a} \mapsto \sigma_a$ is bijective (you need to show injectivity and surjectivity. Using "order considerations" here is circular reasoning, because you have to already know the map is a bijection in order to prove that the sets are the same size).

Handout #2: 9 points.

- (1) Classifying group elements based on the type of action (these will correspond to conjugacy classes in S_4).
- (4 for each problem) For each class, find how many diagrams are fixed, and apply Burnside's Lemma.