MAT347Y1 HW6 Marking Scheme

Friday, October 30

Total: 25 points.

3.4.6: 5 points.

- (1) Base case (note that $G = 1$ isn't allowed by the statement of the theorem. Technically, proving the result for all G does prove it for $G \neq 1$, but you need to be extremely careful: in particular, $1 \trianglelefteq G$ is *not* a composition series, because $G/1$ is not simple), simple groups case
- (1) Choose a nontrivial normal subgroup H of G
- (1) Induction hypothesis gives two composition series
- (1) Lattice isomorphism Theorem
- (1) Third isomorphism theorem

Note: if you choose H to be maximal, you can save yourself a bit of work (only need to look at one composition series), but you still need to prove G/H is simple (this can be done using the lattice isomorphism theorem).

3.5.16:

a) (3 points)

b) (5 points)

Note: the only way many of you found to prove (b) was by computing absurd numbers of elements. Here are some methods that allow you to get away with a lot less:

- If you're willing to use the fact that A_5 is simple: exhibit a 5-cycle and a product of two 2-cycles, conclude that $30 \leq |\langle x, y\rangle|$, and note that simplicity of A_5 means there are no subgroups of index 2.
- Recall question 3 on the October 16 worksheet; in a similar way, you can prove in stages that the alternating group has a small generating set (eventually getting down to $\{x, y\}$).
- Exhibit a 3-cycle that fixes an element in common with x; then use part (a) and the Orbit-Stabilizer theorem (I wasn't aware of this technique until I saw it one of the assignments I was marking, but it's really slick)

4.1.3: 4 points.

4.2.7:

a) (1 point)

b) (3 points)

4.3.22: 4 points.