MAT347Y1 HW8 Marking Scheme

Friday, November 13

Total: 33 points.

5.1.12(a): 6 points.

- (2) A, B isomorphic to image
- (2) intersection isomorphism
- (1) intersection is in the center
- (1) order of the group

5.1.12(b): 3 points. Note that in this case, it's possible to write down presentations of each of the groups in question. It may be easier to define an isomorphism once you write presentations first (so you know where generators need to go).

5.4.5: 4 points.

5.4.14: 5 points. If you apply the theorem for recognizing direct products, you need to prove:

- (1) $G = DU$
- (1) $D \cap U = \{1\}$
- (1) $D \trianglelefteq G$
- (2) $U \trianglelefteq G$. Be careful: U is not equal to $G \cap \mathrm{SL}_n(F)$ (for instance, note that $\left(\begin{smallmatrix} -1 & 0 \\ 0 & -1 \end{smallmatrix}\right)$ is in $G \cap \mathrm{SL}_n(F)$ but not in U). To prove that $gUg^{-1} = U$, use the fact that any $g \in G$ is of the form du for some $d \in D$, $u \in U$, and that elements of D commute with everything.

5.4.19:

(a) 1 point.

(b) 3 points.

(c) 2 points.

(d) 3 points. If you define a subgroup by the join of all perfect subgroups, you need to show that at least one perfect subgroup exists!

5.5.22(a): 4 points.

- (1) $G = UD$
- (1) $D \cap U = \{1\}$
- (2) $U \trianglelefteq G$.

5.5.22(b): 2 points.