MAT 332, Spring 2010. Assignment 4. Due on March 16, in class.

1.

$$\begin{cases} x' = x(1-4x^2-y^2) - 0.5y(1+x) \\ y' = y(1-4x^2-y^2) + 2x(1+x) \end{cases}$$

a) Verify that the origin is an unstable fixed point.

b) Use the Lyapunov function

$$L(x,y) = (1-4x^2-y^2)^2$$

and Poincaré-Bendixson Theorem to show that the system has a limit cycle.
Hint: First show that $\frac{d}{dt}L(x(t),y(t)) < 0$ except at the origin and the ellipse $4x^2+y^2=1$. Then draw contours inside and outside the ellipse, on which $L(x,y)$ is constant and positive.

c) Draw a Maple plot to verify that this cycle is the ellipse

$$4x^2+y^2=1.$$

2. Find an example of a chaotic attractor in 3D which is **NOT** covered in Chapter 7 of the textbook (either in other literature, or on the Internet). Give a brief description of your example, and illustrate its properties with a Maple worksheet (**Caution:** identical answers may be considered cheating).

3. Investigate what happens with the Lorentz system (7.1.1) on p. 245 of the text when parameters are changed. For $\sigma = 10$, $b = 8/3$, and $r = 28$ we observe chaos. With the same values of σ and b, use Maple to determine what happens for $r = 10$, $r = 22$, and $r = 100$. In the last case you will need to start with a very large range of x, y, z values, so the trajectory fits in your plot. To make sure you are seeing the true long-term behaviour of the trajectories, use large values of t (starting at least from $t = 200$).