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NUMBER OF SOLUTIONS WITH A NORM BOUNDED BY A GIVEN CONSTANT OF A SEMILINEAR ELLIPTIC PDE WITH A GENERIC RIGHT-HAND SIDE

ALEXANDER NABUTOVSKY

ABSTRACT. We consider a semilinear boundary value problem $-\Delta u + f(u, x) = 0$ in $\Omega \subset \mathbb{R}^N$ and $u = 0$ on $\partial\Omega$. We assume that f is a C^∞-smooth function and Ω is a bounded domain with a smooth boundary. For any C^α-smooth perturbation $h(x)$ of the right-hand side of the equation we consider the function $N_h(S)$ defined as the number of $C^{2+\alpha}$-smooth solutions u such that $\|u\|_{C^0(\Omega)} \leq S$ of the perturbed problem.

How "small" $N_h(S)$ can be made by a perturbation $h(x)$ such that $\|h\|_{C^0(\Omega)} \leq \varepsilon$? We present here an explicit upper bound in terms of ε, S and

$$\max_{|u|\leq S,\, x\in\overline{\Omega}} \|D_u^i f(u, x)\| \qquad (i \in \{0, 1, 2\}).$$

If S is fixed then h can be chosen by such a way that the upper bound persists under small in C^0-topology perturbations of h. We present an explicit lower bound for the radius of the ball of such admissible perturbations.

1. INTRODUCTION

Let an elliptic problem

$$\begin{cases} -\Delta u + f(u, x) = 0 & \text{in } \Omega, \quad (1.1a) \\ u = 0 & \text{on } \partial\Omega, \quad (1.1b) \end{cases}$$

be given. We assume that $\Omega \subset \mathbb{R}^N$ is a bounded domain with a C^∞-smooth boundary and $f: \mathbb{R} \times \overline{\Omega} \to \mathbb{R}$ is a C^∞-smooth function. We consider this problem in the space $C^{2+\alpha}(\overline{\Omega})$ for some $\alpha \in (0, 1)$.

The set of solutions of the problem (1.1) can be uncountable. However, it follows from Smale results that there exists an arbitrarily small C^α-smooth perturbation of the right-hand side of (1.1a) making the solution set of the problem (1.1) discrete. More precisely it follows from results of [13] that for any $\varepsilon > 0$ there exists a function $h \in C^\alpha(\overline{\Omega})$ such that

(1.2)(1)
$$\max_{x\in\Omega} |h(x)| \leq \varepsilon.$$

(2) All solutions of the problem

$$\begin{cases} -\Delta u + f(u, x) = h(x) & \text{in } \Omega, \quad (1.3a) \\ u|_{\partial\Omega} = 0, & \quad (1.3b) \end{cases}$$

are isolated.

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