MAT137 - Calculus with proofs

- Test 4 opens today
- Assignment 9 due on March 25

- Today: More properties of series

- Monday: Integral test & comparison tests for series
  - **Watch videos 13.10, 13.12**
  - Supplementary video: 13.11

Rapid questions: geometric series

Convergent or divergent?

1. $$\sum_{n=0}^{\infty} \frac{1}{2^n}$$

2. $$\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$$

3. $$\sum_{n=1}^{\infty} \frac{1}{2^{n/2}}$$

4. $$\sum_{n=5}^{\infty} \frac{3^n}{2^{2n+1}}$$

5. $$\sum_{n=3}^{\infty} \frac{3^n}{1000 \cdot 2^{n+2}}$$

6. $$\sum_{n=0}^{\infty} (-1)^n$$

True or False – The Necessary Condition

1. IF $\lim_{n\to\infty} a_n = 0$, THEN $\sum_{n}^{\infty} a_n$ is convergent.

2. IF $\lim_{n\to\infty} a_n \neq 0$, THEN $\sum_{n}^{\infty} a_n$ is divergent.

3. IF $\sum_{n}^{\infty} a_n$ is convergent THEN $\lim_{n\to\infty} a_n = 0$.

4. IF $\sum_{n}^{\infty} a_n$ is divergent THEN $\lim_{n\to\infty} a_n \neq 0$.

What can you conclude?

Assume $\forall n \in \mathbb{N},\ a_n > 0$. Consider the series $\displaystyle\sum_{n=0}^{\infty} a_n$.

Let $\{S_n\}_{n=0}^{\infty}$ be its sequence of partial sums.

In each of the following cases, what can you conclude about the *series*? Is it convergent, divergent, or we do not know?

1. $\forall n \in \mathbb{N}, \quad \exists M \in \mathbb{R}$ s.t. $\quad S_n \leq M$.
2. $\exists M \in \mathbb{R}$ s.t. $\quad \forall n \in \mathbb{N}, \quad S_n \leq M$.
3. $\exists M > 0$ s.t. $\quad \forall n \in \mathbb{N}, \quad a_n \leq M$.
4. $\exists M > 0$ s.t. $\quad \forall n \in \mathbb{N}, \quad a_n \geq M$.

Functions as series

You know that when $|x| < 1$:

$$f(x) = \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

Find similar ways to write the following functions as series:

1. $g(x) = \dfrac{1}{1+x}$
2. $h(x) = \dfrac{1}{1-x^2}$
3. $G(x) = \ln(1+x)$
4. $A(x) = \dfrac{1}{2-x}$

Hint: For (3.) compute G'.