MAT137 - Calculus with proofs

- Assignment 10 due on April 8
- Test 5 opens on April 22

- Today: Analytic functions
- Friday: no class (Good Friday)
- Monday: Constructing new power series **(Videos 14.9, 14.10)**
- After: Applications!

- Please fill out course evaluations

Warm up

1. Write down the Maclaurin series for $f(x) = \sin x$. (Just recall it.)

2. Compute the interval of convergence of this power series.

3. Write down the statement of Lagrange's Remainder Theorem. (Just recall it. Look it up if needed.)

sin is analytic

Let $f(x) = \sin x$. You know its Maclaurin series is

$$S(x) \; = \; \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \; = \; x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

As you know, to prove that $\sin x = S(x)$ we need to show that

$$\forall x \in \mathbb{R}, \quad \lim_{n \to \infty} R_n(x) = 0$$

Use Lagrange's Remainder Theorem to prove it!

Reminder: Lagrange's Remainder Theorem says that given f, a, x, and n with certain conditions,

$$\exists \xi \text{ between } a \text{ and } x \text{ s.t.} \qquad R_n(x) \; = \; \frac{f^{(n+1)}(\xi)}{(n+1)!} (x-a)^{n+1}$$

Generalize your proof

Theorem

Let I be an open interval. Let $a \in I$. Let f be a C^∞ function on I. Let $S(x)$ be the Taylor series for f centered at a.

- IF $\boxed{???}$
- THEN $\forall x \in I, \; f(x) = S(x)$

Which condition can you write instead of "$\boxed{???}$" to make the theorem true?

If you are thinking "the derivatives must be bounded", then you are on the right track, but you need to be much more precise. Which derivatives? On which domain? There are a lot of variables here; can the bounds depend on any variable?