## MAT347Y1 HW11 Marking Scheme

Friday, January 30

## Total: 38 points.

**#1:** 5 points.

#3: 15 points. Note that though there is a nice notion of degree on B, you have to define it if you want to use it (degree has only been defined for actual polynomials so far. And if the set of allowable exponents changes, sometimes degree doesn't behave so nicely)

- (a) 3 points.
- (b) 2 points: Non-zero constants.
- (c) 2 points.
- (d) 2 points. If  $X = p_1 \cdots p_n$ , use part (c) with k > n.
- (e) 2 points. Given  $p, q \in B_N$ , use the isomorphism from part (a) to get a gcd in  $\mathbb{C}[X]$ , and then translate back.
- (f) 2 points. (One example: non-constant polynomials)
- (g) 2 points. If  $p \in B_N$ , use the isomorphism mapping  $B_{N+1}$  to  $\mathbb{C}[X]$ .

**#2:** 18 points.

- (a) 3 points. Note that you have to give two distinct factorizations, show the factors are irreducible, *and* show that they are not associate.
- (b) 1 point.
- (c) 2 points. There are 6.
- (d) 6 points. (1) correct statement iii, (1) equivalence of i and ii (almost identical to  $\mathbb{Z}[i]$  case), (1) iii implies ii (modular arithmetic case checking), (3) i implies iii  $(p \equiv 1 \pmod{3})$  iff there's an element of order 3 in  $(\mathbb{Z}/p\mathbb{Z})^{\times}$  iff  $p \mid (m^2 + m + 1)$ . And don't forget about p = 3)
- (e) 2 points.
- (f) 4 points.