MAT347Y1 HW14 Marking Scheme

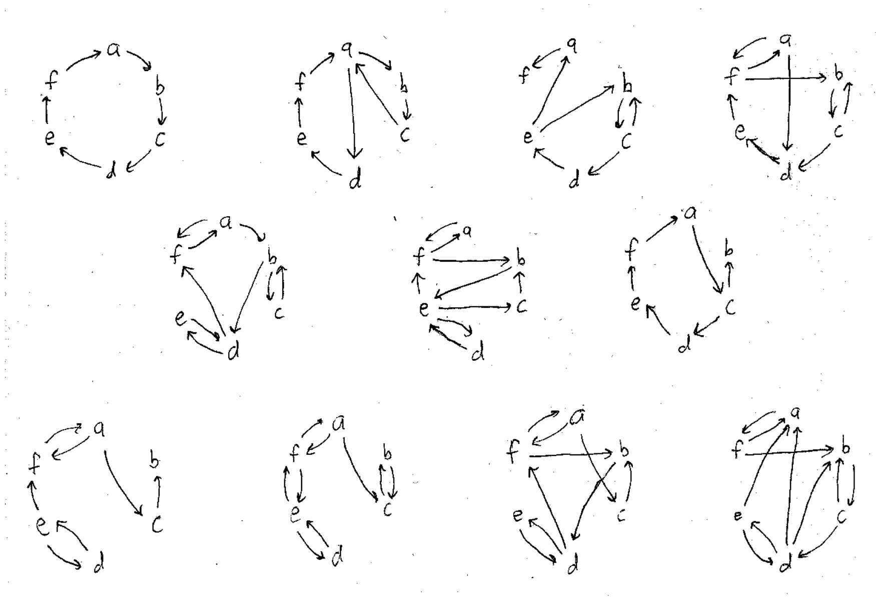
Friday, March 13

Total: 57 points.

#1: 6 points: 2 per part.

#2: 3 points.

#3: 13 points: 2 per implication, +1 for use of Theorems A/B (so essentially one implication is worth 3 points). Note that proving the results in the order given is very straightforward, but many people instead got very creative with the implications they decided to prove. A compilation of everyone's proof diagrams is below: which ones are complete?



#4: 15 points:

- (1) The relation given defines a partial order
- (1) X is nonempty
- (1) Given a chain, defining what should be an upper bound (U_1, U_2, Φ) via unions
- (2) Each U_i is an intermediate field extension (so you need to prove it's a field, contains F, and is contained in K_i)
- Φ is a (1) well-defined (1) injective (1) surjective (1) homomorphism, and (1) it satisfies the desired relation with all isomorphisms in the given chain (so that what you defined is actually an upper bound)

- (1) Apply Zorn's Lemma to obtain a maximal element (L_1, L_2, Ψ).
- Let $i = 1, j = 2$ or $i = 2, j = 1$. If L_i is not all of K_i, choose an element α in $K_i \setminus L_i$, (1) show α has a minimal polynomial f over L_i, and (1) conclude that $\Psi(f)$ (or $\Psi^{-1}(f)$ if $i = 2$) has a root β in $K_j \setminus L_j$. (Note that showing $L_1 \neq K_1$ leads to a contradiction is not enough, because a priori, it could be the case that $L_1 = K_1$ but $L_2 \neq K_1$; (1) make sure to deal in full generality)
- (2) Apply Theorem A to obtain a contradiction

#5: 20 points.

(a) 5 points: (2) $|\mathrm{Emb}(M_2/M_1)|$ = number of distinct roots (prove $\leq$ and $\geq$ separately), (1) $|\mathrm{Emb}(M_2/M_1)| \leq [M_2 : M_1]$, (2) Equality case (not as trivial as it may seem at first - if you're not using the fact that K/F is normal, you're doing it wrong)

(b) 4 points. Note that you can assume M_3/M_2 and M_2/M_1 are simple, because the general result then follows from forming towers of simple extensions (similarly to what you use for (c) and (d)).

(c) 2 points.

(d) 2 points.

(e) 2 points.

(f) 3 points. Note that a product of separable polynomials isn't separable in general.

(g) 2 points.