MAT347Y1 HW15 Marking Scheme

Friday, March 20

Total: 29 points.

14.5.10: 4 points. Many people tried to say that if $\mathbb{Q}(\sqrt[3]{2})$ were a subextension, then S_3 would be a subgroup of $\mathrm{Gal}(\mathbb{Q}(\zeta_n)/\mathbb{Q})$. *This is false.* The Galois correspondence reverses direction: in this context, S_3 would appears as a *quotient*.

14.3.4: 4 points.

Handout #2(a): 8 points.

- (2) set-up for the lemma: $\mathrm{Gal}(L/\mathbb{Q})$ is a 2-group, and the relationship between M and its Galois group. *Make sure you understand how FTGT reverses directions!* M being 2-filtered is NOT the same as $\mathrm{Gal}(M/\mathbb{Q})$ being 2-filtered.
- (1) statement of the lemma: If G is a 2-group and $H \leq G$, then there exist $H = H_0 \leq \cdots \leq H_n = G$ with $|H_i : H_{i-1}| = 2$ for all i.
- (1) G has a normal subgroup K of order 2 (use the centre)
- (3) If $K \leq H$, use induction on $|G|$ to get chain $H/K = H'_0 \leq \cdots \leq H'_n = G/K$ and apply Lattice Isomorphism Theorem
- (1) If $K \not\leq H$, Then $|KH : H| = 2$ and the previous case can be used to obtain a chain above KH.

Handout #2(b): 3 points. The normal closure is a composite of isomorphic copies of K, each copy of K has a tower of quadratic extensions, and the rest is just being careful with notation.

Handout #2(c): 2 points.

Handout #3: 8 points. (Following Dummit and Foote 14.5.14)

- (2) Express η_1 and η_2 by radicals
- (4) Express η'_1, η'_2, η'_1, η'_2 by radicals (in terms of η_1 and η_2)
- (1) Express η''_1 by radicals.
- (1) Show that $\frac{1}{2}\eta''_1 = \cos\frac{2\pi}{17}$.