MAT347Y1 HW1 Marking Scheme

Friday, September 19

Total: 27 points.

1.5.2: 13 points.

- (12) 4 points for each table:
  - (1) At least half of the entries correct
  - (1) Ten or fewer errors
  - (1) Three or fewer errors
  - (1) No errors
- (1) Consistency in how row/column corresponds to first/second term in product

1.6.17: 3 points.

1.7.17: 5 points.

- (1) conjugation by g is a homomorphism
- (1) conjugation by g is injective
- (1) conjugation by g is surjective (instead of proving injectivity and surjectivity separately, you could also just give the inverse to prove it's a bijection. But note that proving injectivity alone is not enough! Can you think of a group G and injective homomorphism $\phi : G \to G$ such that ϕ is not surjective?)
- (1) conjugation by g preserves order of subgroups (because it is a bijection)
- (1) conjugation by g preserves order of elements (because it is an isomorphism)

1.7.21: 6 points. Let G be the group of rigid motions of the cube.

- (2) Find a homomorphism $\phi : G \to S_4$.
- Any two of the following prove that ϕ is a bijection (the third one is implied by the other two because $|S_4| = 24$):
  - (2) $|G| = 24$ (for example by a "where can each vertex go" argument. Listing all the rigid motions you can find is not sufficient)
  - (2) ϕ is injective (this was by far the biggest source of errors. Just because two rigid motions act the same way on pairs, how do you know they act the same way on the vertices *within* the pairs? Could a rigid motion swap vertices within some of the pairs while keeping the pairs themselves in the same place?)
  - (2) ϕ is surjective (e.g. by providing a generating set of S_4 in terms of images of ϕ).