MAT347Y1 HW7 Marking Scheme

Friday, November 14

Total: 27 points.

1: 4 points. Common incorrect answers:

- $\binom{n}{k}$. Take the example $Z_2 \times Z_2$ ($p = 2$, $n = 2$) and find the number of subgroups of order 2 ($k = 1$) to see where your proof breaks down (what subgroups are you missing?)
- $\binom{\frac{p^n-1}{p-1}}{k}$. If you wrote this you get 1 point because you calculated the answer for $k = 1$ correctly. Take the example $Z_2 \times Z_2$ ($p = 2$, $n = 2$) and find the number of subgroups of order 4 ($k = 2$) to see where your proof breaks down (where are you overcounting?)

2: 5 points.

- (2) Notation for the groups involved (are you listing all the elements uniquely, and does your description of the operation correspond to what the operation actually does?)
- (3) Define an isomorphism between the two groups and prove it's an isomorphism

Note: the question is false for $n = 2$. Because of the way the question was phrased, you will not lose marks for ignoring the $n = 2$ case, but 1 bonus point is given for dealing with the exception.

3: 8 points.

- (2) Apply Sylow's Theorem to conclude that there is a normal Sylow p-subgroup, and that all groups are therefore semidirect products of a Sylow p-subgroup and a Sylow 2-subgroup
- (2) correctly identify the two abelian groups
- (4) correctly identify the nonabelian groups (there are 2 or 3, depending on whether $4 \mid p - 1$ or not)

4:

(a) 3 points. Define a map and prove it's an isomorphism.

(b) 1 points. Originally out of 3, but because the question was slightly misleading these are counted as bonus points (there's a subtlety that's actually quite difficult to deal with correctly. If $\beta\phi_1(K)\beta^{-1} = \phi_2(K)$, you need to find an α such that for all $k \in K$, $\beta\phi_1(\alpha(k))\beta^{-1} = \phi_2(k)$; α is not necessarily the identity!)

(c) 3 points.

(d) 3 points. Note that a group can have two isomorphic subgroups with different centralizers, so showing that $C_{G_1}(H)$ and $C_{G_2}(H)$ are not isomorphic does not show G_1 and G_2 are not isomorphic, unless you can prove H is not isomorphic to any other subgroups.