MAT 347
Classification of groups of order p^3
November 7, 2014

1. Classify groups of order 8 up to isomorphism.
   The outcome of this (or any other) classification should include:

   (a) a list of groups of order 8,

   (b) a proof that every group of order 8 is isomorphic to one on the list, and

   (c) a proof that no two different groups in the list are isomorphic to each other.

   Suggestion: Here are some results you already know that may help you in this problem:

   - Lagrange's theorem.
   - Let G be a group. If $x^2 = 1$ for every $x \in G$ then ...
   - Let G be a group and $H \leq G$. If $|G : H|$ ..., then $H \trianglelefteq G$.
   - Let G be a group and let $H, K \leq G$.
     If ..., then HK is a subgroup.
   - Let G be a group and let $H, K \leq G$.
     If..., then the subgroup HK is isomorphic to $H \times K$.
   - Let G be a group and let $H, K \leq G$.
     If..., then the subgroup HK is isomorphic to a semidirect product $H \rtimes K$.
   - Let G be a group whose order is a power of a prime. Then $Z(G)$...
   - Let G be a group. If a is the only element in G of order 2, then ...
   - The classification of elementary abelian p-groups.

2. (Try this only if you finish problem 1.)
   Let p be an odd prime. Classify groups of order p^3 up to isomorphism.