

MAT 347
An example of the FTGT
March 13, 2015

Let $f(X) = X^4 - 2$. Let K be the splitting field of $f(X)$ over \mathbb{Q} . Let $G = \text{Gal}(K/\mathbb{Q})$.

1. Find all roots of $f(X)$ in \mathbb{C} .
2. Find a set of two elements that generate the field extension K/\mathbb{Q} .
3. Calculate $|K : \mathbb{Q}|$.
4. Find a basis for K as a \mathbb{Q} -vector space.
5. List all the elements of G by showing how they act on a set of generators.
6. Find all intermediate field extensions of K/\mathbb{Q} .
7. Which intermediate field extensions are normal? For each one of them, they have to be the splitting field of some polynomial. Find them.
8. Find a primitive element for the field extension K/\mathbb{Q} .