# MAT 347
# Automorphisms
# October 24, 2014

There are two groups whose automorphism group you fully analyzed in homework:

- *Cyclic groups.* $\operatorname{Aut} Z_n \cong (\mathbb{Z}/n\mathbb{Z})^\times$

  Let $Z_n$ be the cyclic group of order $n$ with multiplicative notation. For $\overline{a} \in (\mathbb{Z}/n\mathbb{Z})^\times$, we can define $\varphi_{\overline{a}} \in \operatorname{Aut}(Z_n)$ by $\varphi_{\overline{a}}(x) = x^a$.

  Then the map
  $$\overline{a} \in (\mathbb{Z}/n\mathbb{Z})^\times \;\mapsto\; \varphi_{\overline{a}} \in \operatorname{Aut}(Z_n)$$
  is an isomorphism.

- *Symmetric groups.* If $n \geq 3$ and $n \neq 6$, then $\operatorname{Aut}(S_n) \cong S_n$. Every automorphism is inner (conjugation) and $S_n$ has trivial centre.

  More specifically, for every $\sigma \in S_n$ we can define $C_\sigma \in \operatorname{Aut}(S_n)$ by $C_\sigma(\tau) = \sigma\tau\sigma^{-1}$.

  Then the map
  $$\sigma \in S_n \;\mapsto\; C_\sigma \in \operatorname{Aut}(S_n)$$
  is an isomorphism.

Now for each of the following groups, study its automorphism group. The best possible outcome is that the automorphism group is isomorphic to a group we already know, you can describe the isomorphism, and you can fully describe each element of $\operatorname{Aut}(G)$. If you cannot do this, try to calculate $|\operatorname{Aut}(G)|$ as a first step, or try to at least get a bound on $|\operatorname{Aut}(G)|$. A good strategy is to search for the possible images of a set of generators.

1. $Z_2 \times Z_2$
2. $Z_4 \times Z_3$
3. $Z_4 \times Z_2$
4. $D_8$
5. $Q_8$
6. $Z_5 \times Z_5 \times Z_5$