SOME EXTRA PROBLEMS ON GROUP THEORY

MAT347

1. For every integer n, find how many elements of order n there are in S_5.

2. Obtain a formula for the number of *different* necklaces that can be made with n stones, if we have stones of k different colours.

   Hint: I suggest you try to solve the cases $n = 5$ and $n = 6$ first.

3. For each of the following claims, either give a proof or find a counterexample:

   (a) The relation "being a subgroup of" is transitive; in other words, if $H \leq G$ and $K \leq H$, then $K \leq G$.

   (b) The relation "being a normal subgroup of" is transitive; in other words, if $H \trianglelefteq G$ and $K \trianglelefteq H$, then $K \trianglelefteq G$.

4. Draw the lattice of subgroups of D_{12}. Indicate which subgroups are normal.

   Note: This problem can be a bit tedious, particularly if you are not organized. It is the kind of exercise that is important to do at least once in your learning of group theory. It will give you insight on how subgroups and generators work together, and how do we figure out whether we have found all the subgroups. I suggest you read section 2.5 for examples first. When you turn in your answer, the lattice is enough; you do not need to add explanations.

5. Prove the a subgroup of index 2 is always normal.

6. Find two different decomposition series of D_{12} that do not produce the same composition factors in the same order. How many different decomposition series does D_{12} have?

7. Classify groups of order 56 up to isomorphism. (You may want to use problem 7 on page 165 as a guide. That problem lists all the groups, but does not prove they complete the classification.)