

MAT 1060: Partial Differential Equations I

Assignment 4, October 21 2009

Read Section 3.2 and 3.3, including the proofs of Lemmas 3 and 4 on p. 131, and the statement of Theorem 7. Please hand in to Ehsan on Wednesday, November 4:

- Chapter 3 (p. 162): Problems 2, 3, 4.

Some older copies have a typo in Problem 3a): The PDE should read

$$x_1 u_{x_1} + 2x_2 u_{x_2} + 3u_{x_3} = 3u.$$

- Chapter 3 (p. 162): Problems 5, 6, 7, 10.

In Problem 6, you may restrict yourself to the simpler case where H is smooth, strictly convex, and grows at infinity

$$\lim_{|p| \rightarrow \infty} \frac{H(p)}{|p|} = \infty.$$

In that case, what is the subdifferential $\partial H(q)$? What is its geometric meaning?

To prove the claimed relationship between the subdifferentials $\partial H(p)$ and $\partial L(q)$, first show that

$$H(p) + L(q) \geq p \cdot q \quad \text{for all } p, q \in \mathbb{R}^n,$$

then investigate the equality cases.