Courant problems

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As is known, the decoupling of a quasi-linear system of pde's

$$\frac{\partial u^i}{\partial t} = \sum_{j=1}^{n} A^i_j(u^1, \cdots, u^n) \frac{\partial u^j}{\partial x} \tag{1}$$

into several non-interacting subsystems drastically effects properties of its solutions and the computer time required for its numerical investigation. The paper is devoted to the following two problems posed by Courant [1]:

I. When a given system (1) of first order pde's can be decoupled in some coordinates $v^1, \cdots, v^n$ into k non-interacting subsystems?

II. When a given system (1) can be transformed in some coordinates $v^1, \cdots, v^n$ into a block-diagonal form ?

In the paper we derive the necessary and sufficient conditions for the decoupling of the quasi-linear systems of pde's into k non-interacting subsystems and the necessary and sufficient conditions for the block-diagonalization of such systems with possible interaction between the blocks [2, 3]. Several necessary conditions for the decoupling of systems (1) are found in terms of the specially constructed invariant polynomials.

References

[1] Courant, R., Hilbert, D.: Methods of mathematical physics, II, Interscience Publishers, New York, 1962

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[3] Bogoyavlenskij, O. I.: Block-diagonalizability problem for hydrodynamic type systems. J. of Math. Phys. **47** (2006)