Convenience Store Revisited

I was pleased to receive a response to the Convenience Store Problem (FN, October 17, 2013) from Kerry Skipper. You may recall that the problem was to identify four prices whose sum and whose product were both equal to 7.11. With the prices given in cents, it was a matter of finding four whole numbers a, b, c, d for which $a + b + c + d = 711$ and $a \times b \times c \times d = 2^6 \times 5^6 \times 3^2 \times 79$. Skipper gave the correct answer; the four items cost \$1.20, \$1.25, \$1.50, \$3.16.

As an applied mathematician working in orbital mechanics, his instinct was to turn to "microprocessors to do the heavy lifting". On his first pass, he had the computer loop through all values of a, b, c up to 708 and set $d = 711 - a - b - c$, a task that occupied about a second and required 59,738,358 evaluations. This was shortened essentially by noting that the sum $a+b+c$ could not exceed 710, so that the number of evaluations was more than halved to 25,235,845. Since the order of the numbers did not matter, making the assumption that a, b, c, d were in increasing order cut the number of evaluations to 8,737,655.

However, by using the divisiblity characteristics of the numbers, we can solve the problem without recourse to a computer at all, just having a few possibilities that can be explored by hand. Briefly, here are the steps towards a solutions.

(1) The numbers can be divisible only by the primes 2, 3, 5 and 79. Exactly one of the numbers, say a, is a multiple of 79.

(2) Since $a + b + c + d$ is not a multiple of 5, at most three of them can be divisible by 5. Since 5 appears six times in the product of these three, either all are multiples of $25 = 5^2$ or else one of them is a multiple of $125 = 5^3$.

(3) a cannot be divisible by 25 ($a \geq 79 \times 25 > 711$ cannot be true).

(4) Since no multiple of 79 differs from 711 by a multiple of 25, the numbers b, c, d cannot all be multiples of 25. Thus, we cannot have three numbers that are multiples of 25.

(5) Therefore, one number, say b, is a multiple of 125. Thus a is one of 79, 158, 237, 316, 395, 474, 553, and b is one of 125, 250, 375, 500, 625. We can eliminate 625, because otherwise $a+b$ would be at least $79+625 = 704$ and $c+d$ at most 7, which one can check not to be possible.

With this spadework behind us, it is now a matter of checking through a limited number of possibilities. First we need to eliminate the possibility that two of b, c, d are multiples of 125, so that we are left to investigate the case that b is divisible by 125, and c and d are multiples of 25 and 5. In this case, $711 - a = b + c + d$ is a multiple of 5 and a can only be 316. A little trial and error leads to the answer: $(a, b, c, d) = (316, 125, 150, 120)$.

The layman is apt to think of the computer as a magic answer-producing device, and not have in mind that behind every algorithm is a skilled programmer. The modern is required to do extremely complex tasks, which even with the advanced technology available would be simply impossible without the ingenuity and skill of the programmer who can provide the oversight to ensure that the algorithms are efficient, accurate and pertinent to the task. I conjecture that the failure of politicians to appreciate this led to a lack of time and resources that caused difficulties for such things as the long gun registry, $e-$health and the rollout of Obamacare