MAT188H1F - Linear Algebra - Fall 2017

Solutions to Term Test 1 - September 26, 2017

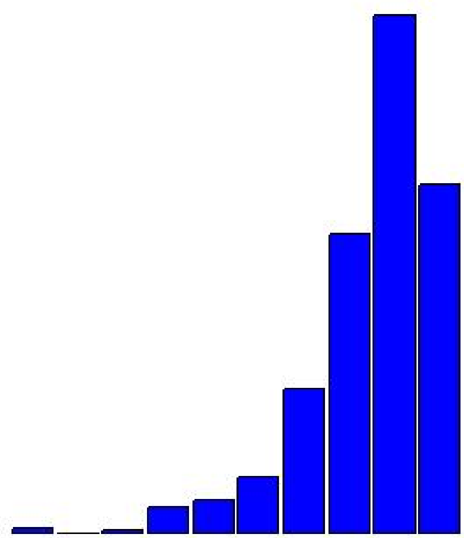
Time allotted: 100 minutes. Aids permitted: Casio FX-991 or Sharp EL-520 calculator.

Comments:

- This test was very straightforward. All the questions were very similar to suggested homework problems or to assigned WeBWorK problems.
- The only question that posed a challenge at all was Question 8, the True or False Question.
- There was (only) one perfect paper.

Breakdown of Results: 836 students wrote this test. The marks ranged from 7.5% to 100%, and the average was 79.3%. Some statistics on grade distribution are in the table on the left, and a histogram of the marks (by decade) is on the right.

Grade	%	Decade	%
		90-100%	24.3%
A	60.4%	80-89%	36.1%
B	20.9%	70-79%	20.9%
C	10.1%	60-69%	10.1%
D	3.9%	50-59%	3.9%
F	4.7%	40-49%	2.3%
		30-39%	1.8%
		20-29%	0.2%
		10-19%	0.0%
		0-9%	0.4%



1. [2 marks for each part; avg: 9.47/10] Find the following:

(a) a unit vector in the direction of $\vec{x} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$.

Solution: $\dfrac{\vec{x}}{\|\vec{x}\|} = \dfrac{1}{\sqrt{4}}\vec{x} = \begin{bmatrix} 1/2 \\ -1/2 \\ 1/2 \\ -1/2 \end{bmatrix}$

(b) the scalar equation of the plane passing through the point $P(4,1,3)$ and orthogonal to the vector $\vec{n} = \begin{bmatrix} 1 \\ 5 \\ -6 \end{bmatrix}$.

Solution: $x_1 + 5x_2 - 6x_3 = 4 + 5 - 18 = -9$.

(c) a vector equation of the line passing through the two points $P(1,3,0,-1)$ and $Q(0,1,1,1)$.

Solution: $\vec{x} = \overrightarrow{OP} + t\,\overrightarrow{PQ} = \begin{bmatrix} 1 \\ 3 \\ 0 \\ -1 \end{bmatrix} + t \begin{bmatrix} -1 \\ -2 \\ 1 \\ 2 \end{bmatrix}$

(d) $\begin{bmatrix} 2 \\ -2 \\ 5 \end{bmatrix} \times \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$

Solution: $\begin{bmatrix} 2 \\ -2 \\ 5 \end{bmatrix} \times \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -6-5 \\ -(6-15) \\ 2+6 \end{bmatrix} = \begin{bmatrix} -11 \\ 9 \\ 8 \end{bmatrix}$

(e) the value(s) of c such that the two vectors $\begin{bmatrix} 10 \\ c \\ 5 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 8 \\ -c \end{bmatrix}$ are orthogonal.

Solution: $\begin{bmatrix} 10 \\ c \\ 5 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 8 \\ -c \end{bmatrix} = 30 + 8c - 5c = 30 + 3c = 0 \Leftrightarrow c = -10.$

2. [avg: 8.1/10] Suppose $\vec{u}$ and $\vec{v}$ are two vectors in $\mathbb{R}^3$ such that the angle between them is $\theta = \dfrac{\pi}{3}$, and $\|\vec{u}\| = 4, \|\vec{v}\| = 7$. Find the exact value of each of the following:

(a) [2 marks] $\vec{u} \cdot \vec{v}$

Solution: $\vec{u} \cdot \vec{v} = \|\vec{u}\|\|\vec{v}\| \cos(\pi/3) = 4(7)(1/2) = 14$

(b) [2 marks] $\|\vec{u} \times \vec{v}\|$

Solution: $\|\vec{u} \times \vec{v}\| = \|\vec{u}\|\|\vec{v}\| \sin(\pi/3) = 4(7)(\sqrt{3}/2) = 14\sqrt{3}$

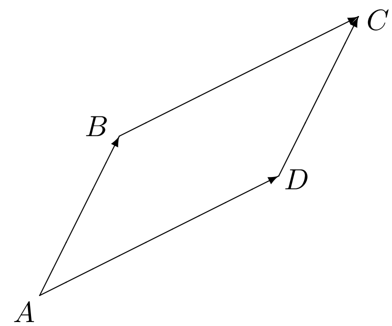
(c) [6 marks] $\|3\,\vec{u} - 5\,\vec{v}\|$

Solution: use $\|\vec{z}\|^2 = \vec{z} \cdot \vec{z}$.

$$\begin{aligned}
\|3\,\vec{u} - 5\,\vec{v}\|^2 &= (3\,\vec{u} - 5\,\vec{v}) \cdot (3\,\vec{u} - 5\,\vec{v}) \\
&= 9\,\vec{u} \cdot \vec{u} - 15\,\vec{u} \cdot \vec{v} - 15\,\vec{v} \cdot \vec{u} + 25\,\vec{v} \cdot \vec{v} \\
&= 9\,\|\vec{u}\|^2 - 30\,\vec{u} \cdot \vec{v} + 25\|\vec{v}\|^2 \\
&= 9(16) - 30(14) + 25(49) = 949 \\
\Rightarrow \|3\,\vec{u} - 5\,\vec{v}\| &= \sqrt{949}
\end{aligned}$$

3. [avg: 8.65/10] Consider the four points $A(1, 3, -1)$, $B(3, 4, 1)$, $C(5, 10, 4)$ and $D(3, 9, 2)$ in $\mathbb{R}^3$.

(a) [4 marks] Show that the four points A, B, C and D are the vertices of a parallelogram.



Solution:

$$\overrightarrow{AB} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \overrightarrow{AD} = \begin{bmatrix} 2 \\ 6 \\ 3 \end{bmatrix}, \overrightarrow{BC} = \begin{bmatrix} 2 \\ 6 \\ 3 \end{bmatrix}, \overrightarrow{DC} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix},$$

so AB and DC are parallel sides, as are AD and BC.

(b) [4 marks] Find the area of the parallelogram determined by A, B, C and D.

Solution:

$$\text{Area } = \|\overrightarrow{AB} \times \overrightarrow{AD}\| = \left\| \begin{bmatrix} 3-12 \\ -(6-4) \\ 12-2 \end{bmatrix} \right\| = \left\| \begin{bmatrix} -9 \\ -2 \\ 10 \end{bmatrix} \right\| = \sqrt{81+4+100} = \sqrt{185}$$

(c) [2 marks] To the nearest degree, find the interior angle at the vertex A of the parallelogram.

Solution: let θ be the angle between $\overrightarrow{AB}$ and $\overrightarrow{AD}$. Then

$$\cos\theta = \frac{\overrightarrow{AB} \cdot \overrightarrow{AD}}{\|\overrightarrow{AB}\|\|\overrightarrow{AD}\|} = \frac{4+6+6}{\sqrt{9}\sqrt{49}} = \frac{16}{21};$$

and to the nearest degree, $\theta = \arccos(16/21) \approx 40°$.

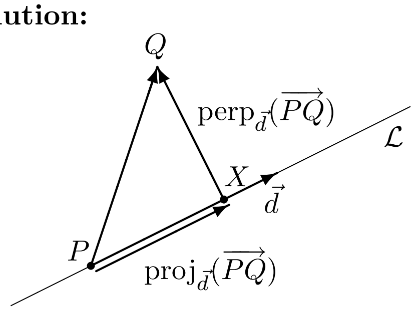
4. [avg: 8.11/10] Let $\mathcal{L}$ be the line with vector equation

$$\vec{x} = \begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix},$$

where t is a parameter. Let Q be the point with coordinates $(-1, 2, 3)$.

(a) [6 marks] Find the point on the line $\mathcal{L}$ that is closest to the point Q.

Solution:



- Let $\vec{d} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ be the direction vector of the line $\mathcal{L}$.
- Let P be the point on $\mathcal{L}$ with coordinates $(4, 1, -2)$.
- Let X be the point on the line $\mathcal{L}$ closest to Q.
- Then $\overrightarrow{PX} = \mathrm{proj}_{\vec{d}}(\overrightarrow{PQ})$.

We have:

$$\overrightarrow{PQ} = \begin{bmatrix} -5 \\ 1 \\ 5 \end{bmatrix}; \ \mathrm{proj}_{\vec{d}}(\overrightarrow{PQ}) = \frac{\overrightarrow{PQ} \cdot \vec{d}}{\vec{d} \cdot \vec{d}} \vec{d} = \frac{-5 + 0 + 10}{1 + 4} \vec{d} = \vec{d}.$$

Consequently the point X on $\mathcal{L}$ closest to Q is given by

$$\begin{bmatrix} x_1 - 4 \\ x_2 - 1 \\ x_3 + 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}.$$

(b) [4 marks] What is the minimum distance from the point Q to the line $\mathcal{L}$?

Solution: let D be the minimum distance from the point Q to the line $\mathcal{L}$. One way to find D is

$$D = \|\overrightarrow{QX}\| = \left\| \begin{bmatrix} 5 + 1 \\ 1 - 2 \\ 0 - 3 \end{bmatrix} \right\| = \sqrt{6^2 + (-1)^2 + (-3)^2} = \sqrt{46}.$$

Alternately, you could use

$$D = \|\mathrm{perp}_{\vec{d}}(\overrightarrow{PQ})\| = \|\overrightarrow{PQ} - \mathrm{proj}_{\vec{d}}(\overrightarrow{PQ})\| = \|\overrightarrow{PQ} - \vec{d}\| = \left\| \begin{bmatrix} -1 - 5 \\ 1 - 0 \\ 5 - 2 \end{bmatrix} \right\| = \sqrt{46}.$$

5. [avg: 8.85/10]

5.(a) [5 marks] Find the scalar equation of the plane that contains the three points $P(5,-1,2)$, $Q(-1,3,4)$ and $R(3,1,1)$.

Solution: we have $\overrightarrow{PQ} = \begin{bmatrix} -1-5 \\ 3+1 \\ 4-2 \end{bmatrix} = \begin{bmatrix} -6 \\ 4 \\ 2 \end{bmatrix}$ and $\overrightarrow{PR} = \begin{bmatrix} 3-5 \\ 1+1 \\ 1-2 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix}$. For the normal to the plane, we can take

$$\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{bmatrix} -6 \\ 4 \\ 2 \end{bmatrix} \times \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -4-4 \\ -(6+4) \\ -12+8 \end{bmatrix} = \begin{bmatrix} -8 \\ -10 \\ -4 \end{bmatrix}.$$

Finally, use either given point, P, Q or R, to find the equation. Using the point P we obtain

$$-8x_1 - 10x_2 - 4x_3 = -8(5) - 10(-1) - 4(2) = -38 \Leftrightarrow 4x_1 + 5x_2 + 2x_3 = 19.$$

5.(b) [5 marks] Find a vector equation of the line of intersection of the two planes with scalar equations $x_1 - 3x_2 - 2x_3 = 10$ and $3x_1 + 2x_2 + x_3 = 2$.

Solution: let $\vec{d}$ be a direction vector of the line; let $\vec{n}_1$ be the normal vector to the plane with equation $x_1 - 3x_2 - 2x_3 = 10$; let $\vec{n}_2$ be the normal vector to the plane with equation $3x_1 + 2x_2 + x_3 = 2$. We have

$$\vec{n}_1 = \begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix} \text{ and } \vec{n}_2 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}.$$

Since the line of intersection of the two planes must be contained in each plane we know $\vec{d} \cdot \vec{n}_1 = 0$ and $\vec{d} \cdot \vec{n}_2 = 0$. Thus we can take

$$\vec{d} = \vec{n}_1 \times \vec{n}_2 = \begin{bmatrix} -3+4 \\ -(1+6) \\ 2+9 \end{bmatrix} = \begin{bmatrix} 1 \\ -7 \\ 11 \end{bmatrix}.$$

To find a point on the line we need to find *any* point of intersection of the two planes. For simplicity, let $x_2 = 0$. Then x_1, x_3 must satisfy $x_1 - 2x_3 = 10$ and $3x_1 + x_3 = 2$, for which the solution is $x_1 = 2, x_3 = -4$. Thus a vector equation of the line is

$$\vec{x} = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix} + t \begin{bmatrix} 1 \\ -7 \\ 11 \end{bmatrix},$$

where t is a parameter.

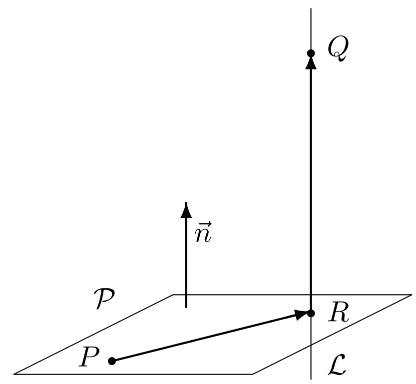
6. [avg: 8.18/10] Let $\mathcal{P}$ be the plane with scalar equation $3x_1 - 4x_2 - 5x_3 = 6$ and Q the point with coordinates $(2, 5, 4)$. Find both

(a) [5 marks] the minimum distance from the point Q to the plane $\mathcal{P}$, and

(b) [5 marks] the point on the plane $\mathcal{P}$ that is closest to the point Q.

You can solve parts (a) and (b) in any order you prefer.

Solution: setting things up.



- The normal vector to $\mathcal{P}$ is $\vec{n} = \begin{bmatrix} 3 \\ -4 \\ -5 \end{bmatrix}$.
- Let R be the point on $\mathcal{P}$ that is closest to Q.
- Suppose P is any point on the plane $\mathcal{P}$.
- Then

$$\overrightarrow{RQ} = \text{proj}_{\vec{n}}\left(\overrightarrow{PQ}\right).$$

For calculations, pick the point P to be $(2, 0, 0)$. (But any point on $\mathcal{P}$ would do.)

(a) Then the minimum distance from Q to the plane is given by $D = \|\overrightarrow{RQ}\| = \|\text{proj}_{\vec{n}}\left(\overrightarrow{PQ}\right)\|$. We have

$$\overrightarrow{PQ} = \begin{bmatrix} 2-2 \\ 5-0 \\ 4-0 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 4 \end{bmatrix} \text{ and } \text{proj}_{\vec{n}}\left(\overrightarrow{PQ}\right) = \frac{0-20-20}{9+16+25}\begin{bmatrix} 3 \\ -4 \\ -5 \end{bmatrix} = -\frac{4}{5}\begin{bmatrix} 3 \\ -4 \\ -5 \end{bmatrix}.$$

Then

$$D = \|\text{proj}_{\vec{n}}\left(\overrightarrow{PQ}\right)\| = \left\| -\frac{4}{5}\begin{bmatrix} 3 \\ -4 \\ -5 \end{bmatrix} \right\| = \frac{4}{5}\sqrt{9+16+25} = 4\sqrt{2}.$$

(b) Let R have coordinates (r_1, r_2, r_3). Then $\overrightarrow{QR} = -\text{proj}_{\vec{n}}\left(\overrightarrow{PQ}\right)$, so that

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix} + \frac{4}{5}\begin{bmatrix} 3 \\ -4 \\ -5 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 22 \\ 9 \\ 0 \end{bmatrix} = \begin{bmatrix} 4.4 \\ 1.8 \\ 0 \end{bmatrix}.$$

See Page 10 for an easier, **Alternate Solution** to this question.

7. [avg: 6.57/10]

7.(a) [4 marks] Let $\vec{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\vec{x} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$. By direct calculation, show that $\text{perp}_{\vec{u}}\left(\text{proj}_{\vec{u}}(\vec{x})\right) = \vec{0}$.

Solution: $\text{proj}_{\vec{u}}(\vec{x}) = \dfrac{\vec{x}\cdot\vec{u}}{\vec{u}\cdot\vec{u}}\vec{u} = \dfrac{2+3-1}{1+1+1}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \dfrac{4}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Then

$$\text{perp}_{\vec{u}}\left(\frac{4}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right) = \frac{4}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \text{proj}_{\vec{u}}\left(\frac{4}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right) = \frac{4}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \frac{4}{3}\left(\frac{3}{3}\right)\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

7.(b) [4 marks] For any non-zero vector $\vec{u} \in \mathbb{R}^3$, prove algebraically that for any $\vec{x} \in \mathbb{R}^3$,

$$\text{perp}_{\vec{u}}\left(\text{proj}_{\vec{u}}(\vec{x})\right) = \vec{0}.$$

Solution: $\text{perp}_{\vec{u}}\left(\text{proj}_{\vec{u}}(\vec{x})\right) = \text{proj}_{\vec{u}}(\vec{x}) - \text{proj}_{\vec{u}}\left(\text{proj}_{\vec{u}}(\vec{x})\right) = \text{proj}_{\vec{u}}(\vec{x}) - \text{proj}_{\vec{u}}(\vec{x}) = \vec{0}$,

where we have made use of the **projection property** of the projection map:

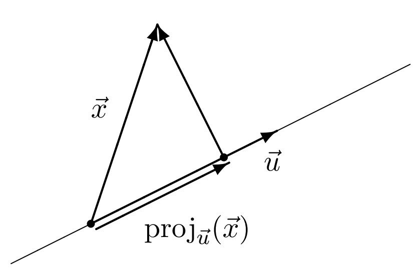
$$\text{proj}_{\vec{u}}\left(\text{proj}_{\vec{u}}(\vec{x})\right) = \text{proj}_{\vec{u}}(\vec{x}).$$

Or, you can prove this last part directly:

$$\text{proj}_{\vec{u}}\left(\text{proj}_{\vec{u}}(\vec{x})\right) = \text{proj}_{\vec{u}}\left(\frac{\vec{x}\cdot\vec{u}}{\|\vec{u}\|^2}\vec{u}\right) = \frac{\left(\frac{\vec{x}\cdot\vec{u}}{\|\vec{u}\|^2}(\vec{u}\cdot\vec{u})\right)}{\|\vec{u}\|^2}\vec{u} = \frac{x\cdot\vec{u}}{\|\vec{u}\|^2}\vec{u} = \text{proj}_{\vec{u}}(\vec{x})$$

7.(c) [2 marks] For $\vec{u} \neq \vec{0}$, explain geometrically why $\text{perp}_{\vec{u}}\left(\text{proj}_{\vec{u}}(\vec{x})\right) = \vec{0}$, for every vector $\vec{x} \in \mathbb{R}^3$. (Draw a diagram.)

Solution:



Since $\text{proj}_{\vec{u}}(\vec{x})$ is parallel to $\vec{u}$, its perpendicular part will be $\vec{0}$; that is,

$$\text{perp}_{\vec{u}}\left(\text{proj}_{\vec{u}}(\vec{x})\right) = \vec{0}.$$

8. [avg: 5.54/10] Indicate if the following statements are **True** or **False**, and give a *brief* explanation why.

(a) [2 marks] If $\vec{u}, \vec{v}, \vec{w}$ are non-zero vectors in $\mathbb{R}^3$ and $\vec{u} \cdot \vec{v} = \vec{u} \cdot \vec{w}$, then $\vec{v} = \vec{w}$. ○ **True** ⊗ **False**

Solution: e.g. $\vec{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, then $\vec{u} \cdot \vec{v} = \vec{u} \cdot \vec{w} = 0$, but $\vec{v} \neq \vec{w}$.

(b) [2 marks] If $\vec{u}, \vec{v}, \vec{w}$ are non-zero vectors in $\mathbb{R}^3$ and $\vec{u} \times \vec{v} = \vec{u} \times \vec{w}$, then $\vec{v} = \vec{w}$. ○ **True** ⊗ **False**

Solution: e.g. $\vec{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$, then $\vec{u} \times \vec{v} = \vec{u} \times \vec{w} = \vec{0}$, but $\vec{v} \neq \vec{w}$.

(c) [2 marks] If $\vec{u}, \vec{v}$ are non-zero vectors in $\mathbb{R}^3$ and $\|\vec{u} \times \vec{v}\| = \|\vec{u}\|\|\vec{v}\|$, then $\vec{u} \cdot \vec{v} = 0$.

⊗ **True** ○ **False**

Solution: since $\|\vec{u} \times \vec{v}\| = \|\vec{u}\|\|\vec{v}\| \sin\theta$, we have $\sin\theta = 1$. Therefore $\theta = \pi/2$, and $\vec{u} \cdot \vec{v} = 0$.

(d) [2 marks] For any two points P, Q in $\mathbb{R}^n$, and any non-zero vector $\vec{u} \in \mathbb{R}^n$,

$$\overrightarrow{OP} + \text{perp}_{\vec{u}}(\overrightarrow{PQ}) = \overrightarrow{OQ} + \text{proj}_{\vec{u}}(\overrightarrow{QP}).$$

⊗ **True** ○ **False**

Solution:

$$\overrightarrow{OP} + \text{perp}_{\vec{u}}(\overrightarrow{PQ}) = \overrightarrow{OP} + \overrightarrow{PQ} - \text{proj}_{\vec{u}}(\overrightarrow{PQ}) = \overrightarrow{OQ} - \text{proj}_{\vec{u}}(-\overrightarrow{QP}) = \overrightarrow{OQ} + \text{proj}_{\vec{u}}(\overrightarrow{QP})$$

(e) [2 marks] Suppose $\mathcal{L}$ is the line in $\mathbb{R}^3$ passing through the origin and parallel to the vector $\vec{u} \neq \vec{0}$. Then the equation

$$\text{proj}_{\vec{u}}(\vec{x}) = \text{proj}_{\vec{u}}(\overrightarrow{OP})$$

is an equation of the line passing through the point P and orthogonal to the line $\mathcal{L}$.

○ **True** ⊗ **False**

Solution: simplify the equation.

$$\text{proj}_{\vec{u}}(\vec{x}) = \text{proj}_{\vec{u}}(\overrightarrow{OP}) \Leftrightarrow \frac{\vec{x} \cdot \vec{u}}{\|\vec{u}\|^2}\vec{u} = \frac{\overrightarrow{OP} \cdot \vec{u}}{\|\vec{u}\|^2}\vec{u} \Leftrightarrow \vec{x} \cdot \vec{u} = \overrightarrow{OP} \cdot \vec{u},$$

which is the equation of a **plane** in $\mathbb{R}^3$ passing through the point P, with normal vector $\vec{u}$.

Alternate Solution to Question 6: let $\mathcal{L}$ be the line normal to the plane, passing through the point Q. A vector equation of this line is

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 4 \end{bmatrix} + t \begin{bmatrix} 3 \\ -4 \\ -5 \end{bmatrix},$$

for which the parametric equations are

$$x_1 = 2 + 3t,\ x_2 = 5 - 4t,\ x_3 = 4 - 5t.$$

Substitute these into the equation of the plane to obtain the value of t for the intersection point of the line and plane:

$$3(2 + 3t) - 4(5 - 4t) - 5(4 - 5t) = 6 \Leftrightarrow t = \frac{4}{5}.$$

Thus:

(b) the point R on the plane closest to Q has coordinates $\left(\frac{22}{5}, \frac{9}{5}, 0\right)$.

(a) and the minimum distance from the plane $\mathcal{P}$ to the point Q is

$$\|\overrightarrow{RQ}\| = \left\| \begin{bmatrix} 2 - 22/5 \\ 5 - 9/5 \\ 4 \end{bmatrix} \right\| = \left\| \frac{1}{5} \begin{bmatrix} -12 \\ 16 \\ 20 \end{bmatrix} \right\| = \frac{4}{5} \left\| \begin{bmatrix} -3 \\ 4 \\ 5 \end{bmatrix} \right\| = \frac{4}{5}\sqrt{50} = 4\sqrt{2},$$

as before.

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