MAT137Y1 – LEC0501

Calculus!

Limits

September 26th, 2018

For next week

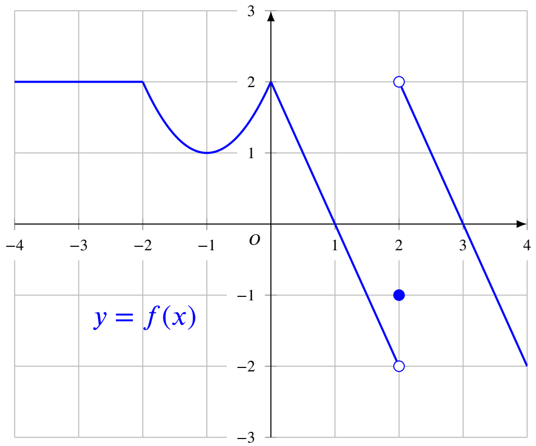
For Monday (Oct 1), watch the videos:

- Proof using the definition of limits: 2.7, 2.8, 2.9

For Wednesday (Oct 3), watch the videos:

- Limit laws: 2.10, 2.11, 2.12, 2.13

Limits from a graph



Find the value of

1. $\lim_{x \to 2} f(x)$
2. $\lim_{x \to 0} f(f(x))$
3. $\lim_{x \to 2} (f(x))^2$
4. $\lim_{x \to 0} f(-2\cos(x))$

Floor

Given a real number x, we defined the *floor of* x, denoted by $\lfloor x \rfloor$, as the largest integer smaller than or equal to x. For example:

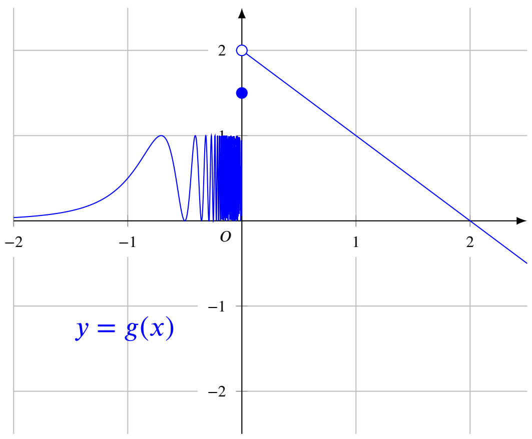
$$\lfloor \pi \rfloor = 3, \qquad \lfloor 7 \rfloor = 7, \qquad \lfloor -0.5 \rfloor = -1.$$

Sketch the graph of $y = \lfloor x \rfloor$.

Then compute:

1. $\lim_{x \to 0^+} \lfloor x \rfloor$
2. $\lim_{x \to 0^-} \lfloor x \rfloor$
3. $\lim_{x \to 0} \lfloor x \rfloor$
4. $\lim_{x \to 0} \lfloor x^2 \rfloor$

More limits from a graph



Find the value of

1. $\lim_{x\to 0^+} g(x)$
2. $\lim_{x\to 0^+} \lfloor g(x) \rfloor$
3. $\lim_{x\to 0^+} g\left(\lfloor x \rfloor\right)$
4. $\lim_{x\to 0^-} g(x)$
5. $\lim_{x\to 0^-} \lfloor g(x) \rfloor$
6. $\lim_{x\to 0^-} \left\lfloor \frac{g(x)}{2} \right\rfloor$
7. $\lim_{x\to 0^-} g\left(\lfloor x \rfloor\right)$

Definitions

Write down the formal definition of the following statements:

1. $\lim_{x\to a} f(x) = L$
2. $\lim_{x\to a} f(x)$ exists
3. $\lim_{x\to a} f(x)$ doesn't exist

Compute a limit

Compute the following limits, or explain why they don't exist:

1. $\lim_{x\to \frac{\pi}{2}} \frac{\cos(x)}{x}$
2. $\lim_{x\to 2} \frac{x^2-4}{x^2-3x+2}$
3. $\lim_{x\to 0} \frac{x^2+2|x|}{x}$
4. $\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

Compute a limit at ∞

Compute the following limits:

1. $\lim_{x\to\infty} \sqrt{x+5}-\sqrt{x-3}$
2. $\lim_{x\to\infty} \frac{x^2+11}{x-3}$
3. $\lim_{x\to\infty} \frac{x^2+2x+1}{3x^2-x-2}$
4. $\lim_{x\to\infty} \sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}$

Limit of exponentials

Compute:

$$\lim_{t\to 0^+} e^{1/t}, \qquad \lim_{t\to 0^-} e^{1/t}.$$

Suggestion: sketch the graph of $y = e^x$ first.

Is it correct?

We want to compute $\lim_{x\to-\infty}\left(x-\sqrt{x^2+x}\right)$.

$$\lim_{x\to-\infty}\frac{\left(x-\sqrt{x^2+x}\right)\left(x+\sqrt{x^2+x}\right)}{x+\sqrt{x^2+x}} = \lim_{x\to-\infty}\frac{x^2-(x^2+x)}{x+\sqrt{x^2+x}}$$

$$= \lim_{x\to-\infty}\frac{-x}{x\left(1+\sqrt{1+\frac{1}{x}}\right)} = \lim_{x\to-\infty}\frac{-1}{1+\sqrt{1+\frac{1}{x}}} = -\frac{1}{2}$$