Integration by substitution

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Computation practice: integration by substitution

Use substitutions to compute:

1. $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$
2. $\displaystyle\int e^x \cos\left(e^x\right) dx$
3. $\displaystyle\int \cot x \, dx$
4. $\displaystyle\int x e^{-x^2} dx$
5. $\displaystyle\int \frac{e^{2x}}{\sqrt{e^x+1}} dx$

Definite integral via substitution

This final answer is right, but the write-up is WRONG. Why?

Compute $I = \int_0^2 \sqrt{x^3+1}\, x^2 dx$

Wrong answer

Substitution: $u = x^3 + 1,\ du = 3x^2 dx$.

$$
\begin{aligned}
I &= \frac{1}{3} \int_0^2 \sqrt{x^3+1}\,(3x^2 dx) && = \frac{1}{3} \int_0^2 u^{1/2}\, du \\
&= \frac{1}{3}\,\frac{2}{3}\ u^{3/2}\Big|_0^2 && = \frac{1}{9}\left(x^3+1\right)^{2/3}\Big|_0^2 \\
&= \frac{2}{9}\left(2^3+1\right)^{3/2} - \frac{2}{9}\,(0+1)^{3/2} && = \frac{52}{9}
\end{aligned}
$$

A different kind of substitution

Compute

$$\int_0^1 \sqrt{1-x^2}\,dx$$

using the substitution $x = \sin\theta$.