pg. 21 6. (b) (i) Solution for x, y, z in terms of w. Augmented matrix for the system is

$$\begin{array}{cccc|c} x & y & z & w & \\ \textcircled{1} & 1 & 2 & 1 & 4 \\ 2 & -2 & 3 & -2 & 5 \\ 1 & 7 & 3 & 5 & 7 \end{array} \simeq \begin{array}{cccc|c} x & y & z & w & \\ 1 & 1 & 2 & 1 & 4 \\ 0 & -4 & -1 & -4 & -3 \\ 0 & 6 & \textcircled{1} & 4 & 3 \end{array}$$

$$\simeq \left[\begin{array}{cccc|c} 1 & -11 & 0 & -7 & -2 \\ 0 & \textcircled{2} & 0 & 0 & 0 \\ 0 & 6 & 1 & 4 & 3 \end{array}\right] \simeq \left[\begin{array}{cccc|c} 1 & 0 & 0 & -7 & -2 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 & 3 \end{array}\right]$$

Solution is $x = 7w - 2$, $y = 0$, $z = -4w + 3$.

(ii) Solution for X, Z, W in terms of y. Following the above sequence until the third matrix, one would pivot as indicated:

$$\begin{array}{cccc|c} x & y & z & w & \\ 1 & -11 & 0 & -7 & -2 \\ 0 & 2 & 0 & \textcircled{0} & 0 \\ 0 & 6 & 1 & 4 & 3 \end{array}.$$

But "0" is not an admissible pivot (division by 0 is not possible), so there is no solution for x, z, w in terms of y. Alternatively, from (i), in any solution of the system, y remains at a constant value, regardless of the values of the other three variables. Note that the "x", "z", and "w" columns of the first augmented matrix are linearly dependent (

$$7 \times \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + (-4) \times \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix} + (1) \cdot \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

is a relation of linear dependence) so that $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -2 \\ 1 & 3 & 5 \end{bmatrix}$ is not invertible and the system $x + 2z + w = 4 - y$, $2x + 3z - 2w = 5 + 2y$, $x + 3z + 5w = 7 - 7y$ does not have

<u>pg. 21 6. (b) (ii) (cont'd.)</u> a unique solution. That is, there is no formula which gives x, z, and w in terms of y. In fact, from (i) this system <u>has</u> a solution only if $y = 0$.

<u>pg. 21 9. (a)</u> Augmented matrix is

$$\left[\begin{array}{cccc|c} x & y & z & w & \\ 2 & 1 & 2 & 1 & 2 \\ \textcircled{1} & 3 & -2 & -3 & -4 \\ 4 & 2 & 1 & 0 & 2 \\ -2 & -6 & 1 & 4 & 2 \end{array}\right] \simeq \left[\begin{array}{cccc|c} x & y & z & w & \\ 0 & -5 & 6 & 7 & 10 \\ 1 & 3 & -2 & -3 & -4 \\ 0 & -10 & 9 & 12 & 18 \\ 0 & 0 & -3 & \textcircled{-2} & -6 \end{array}\right]$$

$$\simeq \left[\begin{array}{cccc|c} 0 & \textcircled{-5} & -\frac{9}{2} & 0 & -11 \\ 1 & 3 & \frac{5}{2} & 0 & 5 \\ 0 & -10 & -9 & 0 & -18 \\ 0 & 0 & \frac{3}{2} & 1 & 3 \end{array}\right] \simeq \left[\begin{array}{cccc|c} 0 & 1 & \frac{9}{10} & 0 & \frac{11}{5} \\ 1 & 0 & -\frac{1}{5} & 0 & -\frac{18}{5} \\ 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & \frac{3}{2} & 1 & 3 \end{array}\right]$$

The given system has the same solution set as the system $y + \frac{9}{10} z = \frac{11}{5}$, ie, no solutions.

$$x - \frac{1}{5} z = -\frac{18}{5}$$

$$0 = 3 \longleftarrow !$$

$$\frac{3}{2} z + w = 3$$

<u>pg. 28 6. (c)</u> Augmented matrix for inversion is

$$\left[\begin{array}{ccc|ccc} 2 & \textcircled{1} & 3 & 1 & 0 & 0 \\ 4 & 6 & 2 & 0 & 1 & 0 \\ -1 & -6 & 4 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 2 & 1 & 3 & 1 & 0 & 0 \\ \textcircled{-8} & 0 & -16 & -6 & 1 & 0 \\ 11 & 0 & 22 & 6 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 0 & 1 & -1 & -\frac{1}{2} & \frac{1}{4} & 0 \\ 1 & 0 & 2 & \frac{3}{4} & -\frac{1}{8} & 0 \\ 0 & 0 & 0 & -\frac{9}{4} & \frac{11}{8} & 1 \end{array}\right]$$; the given matrix is not invertible.

pg. 28 8. (b) Augmented matrix for inversion is

$$\left[\begin{array}{ccc|ccc} \textcircled{1} & 2 & 1 & 1 & 0 & 0 \\ 1 & 3 & -1 & 0 & 1 & 0 \\ 1 & 2 & 2 & 0 & 0 & 1 \end{array}\right] \simeq \left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ 0 & 0 & \textcircled{1} & -1 & 0 & 1 \end{array}\right]$$

$$\simeq \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 2 & 0 & -1 \\ 0 & \textcircled{1} & 0 & -3 & 1 & 2 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right] \simeq \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & -2 & -5 \\ 0 & 1 & 0 & -3 & 1 & 2 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

inverse of given matrix

pg. 42 5. (d) We seek a relation of linear dependence for the three vectors, ie, a non-trivial solution of the system represented by the augmented matrix (in which c_1, c_2, c_3 are the sought for coefficients):

$$\begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & \\ \end{array} \\ \left[\begin{array}{ccc|c} 2 & 4 & 1 & 0 \\ \textcircled{1} & 5 & 2 & 0 \\ 3 & 1 & -1 & 0 \end{array}\right] \end{array} \simeq \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & \\ \end{array} \\ \left[\begin{array}{ccc|c} 0 & -6 & \textcircled{-3} & 0 \\ 1 & 5 & 2 & 0 \\ 0 & -14 & -7 & 0 \end{array}\right] \end{array} \simeq \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & \\ \end{array} \\ \left[\begin{array}{ccc|c} 0 & 2 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

Any non-trivial solution of the system $\begin{array}{l} 2c_2 + c_3 = 0 \\ c_1 + c_2 \quad = 0 \end{array}$ will do.

We take $c_1 = 1$, $c_2 = -1$, $c_3 = 2$.

$$\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} - \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

is a relation of linear dependence for the given vectors (which may be used to express any of the three vectors as a linear combination of the other two).

pg. 42 6. (b) We follow the same plan as in the previous question.

$$\begin{array}{cccc|c} c_1 & c_2 & c_3 & c_4 & \\ 2 & 0 & 8 & 3 & 0 \\ \enclose{circle}{1} & 0 & 6 & 2 & 0 \\ 1 & 1 & 4 & 0 & 0 \\ 1 & 2 & 6 & 1 & 0 \end{array} \simeq \begin{array}{cccc|c} c_1 & c_2 & c_3 & c_4 & \\ 0 & 0 & -4 & -1 & 0 \\ 1 & 0 & 6 & 2 & 0 \\ 0 & \enclose{circle}{1} & -2 & -2 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array}$$

$$\simeq \begin{array}{cccc|c} c_1 & c_2 & c_3 & c_4 & \\ 0 & 0 & -4 & \enclose{circle}{-1} & 0 \\ 1 & 0 & 6 & 2 & 0 \\ 0 & 1 & -2 & -2 & 0 \\ 0 & 0 & 4 & 3 & 0 \end{array} \simeq \begin{array}{cccc|c} c_1 & c_2 & c_3 & c_4 & \\ 0 & 0 & 4 & 1 & 0 \\ 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 6 & 0 & 0 \\ 0 & 0 & \enclose{circle}{-8} & 0 & 0 \end{array} \simeq \begin{array}{cccc|c} c_1 & c_2 & c_3 & c_4 & \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array}$$

This last matrix represents the system

$$\begin{array}{l} c_4 = 0 \\ c_1 = 0, \\ c_2 = 0 \\ c_3 = 0 \end{array}$$

which has the same solution set as the system which is represented by the first matrix in the sequence.

Thus, the only linear combination of the four given vectors which totals $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, has all of its coefficients equal to 0; the four given vectors are linearly independent.