2015-09-29

Apm236h1f 2015

Problems 3

1. Let f,g be two convex functions from R^n to R. Let c be a real number. Show that:

(a) f+g is convex.

(b) If c>0, then cf is convex.

2. Convert the following problems to linear programming problems (l.p.p.'s):

(a) minimize $2x+3|y-10|$
subject to $x+y<=3$

(b) minimize $\max\{-x,1,x\}$
subject to $x>=5$

(c) minimize the largest residual (a residual is the prediction error at the ith data point) given the 3 data points A=(3/4,1), B=(7/4,3), C=(11/4,3), and assuming that your function is linear (that is of the form y=mx). (At the end, make sure to state your decision variables).

3. Solve the following lpp's.

(a)

Maximize $z=x+y$
subject to $2x+y<=8$
$x+3y<=9$
$x>=0$, $y>=0$

(b) Same problem as in (a), but Minimize.

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Problems 3 solutions

① (a) $(f+g)(tx+(1-t)y)$

$= f(tx+(1-t)y) + g(tx+(1-t)y)$

$\leq tf(x) + (1-t)f(y) + tg(x) + (1-t)g(y)$

$= t(f+g)(x) + (1-t)(f+g)(y)$.

(b) $cf(tx+(1-t)y)$

$\leq ctf(x) + c(1-t)f(y)$, since $c>0$

$= tcf(x) + (1-t)cf(y)$.

② (a) Let "z" represent $|y-10|$.

Then, we get

minimize $2x+3z$

subject to $z \geq y-10$

$z \geq 10-y$

$x+y \leq 3$

w/ decision variables x, y, z.

(b) Let z represent $\max\{-x, 1, x\}$

Then, we get

minimize z

subject to $z \geq -x$

$z \geq 1$

$z \geq x$

$x \geq 5$.

w/ decision variables x, z

(c) minimize $\max\{ |1-\frac{3}{4}m|, |3-\frac{7}{4}m|, |3-\frac{11}{4}m| \}$

subject to no constraints.

w/ decision variable m.

becomes.

minimize Z

subject to

$$Z \geq |1-\tfrac{3}{4}m|$$
$$Z \geq |3-\tfrac{7}{4}m|$$
$$Z \geq |3-\tfrac{11}{4}m|$$

which becomes

minimize Z

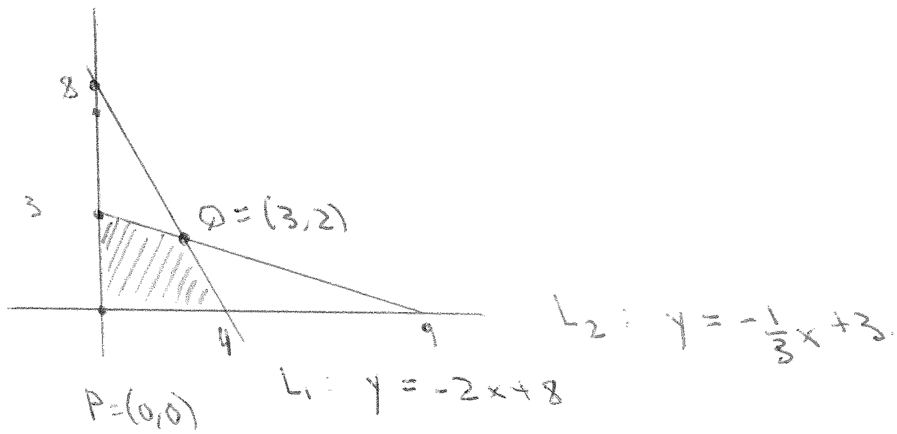
subject to

$$\begin{aligned} Z &\geq 1-3/4\,m \\ Z &\geq +3/4\,m-1 \\ Z &\geq 3-\tfrac{7}{4}m \\ Z &\geq \tfrac{7}{4}m-3 \\ Z &\geq 3-\tfrac{11}{4}m \\ Z &\geq \tfrac{11}{4}m-3. \end{aligned}$$

w/ decision variables m, Z.

(3) $\vec{c} = (1,1)$.

Draw picture:



For (a),

to maximize, move in direction of $\vec{c}$,
so $Q = (3,2)$ is optimal.

For (b)

to minimize, move in direction of $-\vec{c}$
so $P = (0,0)$ is optimal.