Hmwk 2.1

$\max \langle \hat{c}, \hat{x} \rangle$, $\hat{c}' = (2, 1, 6, -4)$.

is "equivalent to" $\min \langle -\hat{c}, \hat{x} \rangle$

i.e Let $\hat{c}' = (-2, -1, -6, +4)$

Now, w/ slack/surplus var's $x_5, x_6, x_7 \geq 0$

we get: w/ $x' = (x_1, x_2, x_3, x_4, x_5, x_6, x_7) \geq 0$

and $c' = (-2, -1, -6, +4, 0, 0, 0)$.

$b' = (6, 12, 2)$.

$$\begin{array}{cc} & \begin{array}{ccccccc} \text{basic} & & & & & & \\ \downarrow & \downarrow & & \downarrow & & & \end{array} \\ A = & \begin{bmatrix} 1 & 2 & 4 & -1 & 1 & 0 & 0 \\ 2 & 3 & -1 & 1 & 0 & 1 & 0 \\ 1 & 0 & +1 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Check that A_1, A_2, A_4 are indeed l-ind.

Thus, since $m = 3$ (A has full row rank),

$$B = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Find B^{-1} and you should get:

$$B^{-1} = (1/4) \cdot \begin{bmatrix} 3 & -2 & 5 \\ -1 & 2 & -3 \\ -3 & 2 & -1 \end{bmatrix}.$$

Notice that:

$$x_B' = [B^{-1} b]' = \frac{1}{8}(2, 6, 2) = \frac{1}{4}(1, 3, 1).$$

i.e. $x_1 = 1/4$, $x_2 = 3/4$, $x_4 = 1/4$. $c_B' = (-2, -1, 4)$

i.e. $x' = \left(\frac{1}{4}, \frac{3}{4}, 0, \frac{1}{4}, 0, 0, 0\right)$. Let $p' = c_B' B^{-1}$

non-basic

Thus $p' = \frac{1}{4}(-17, 10, -11)$.

Now $p' \left[A_3 \, A_5 \, A_6 \, A_7 \right] = \frac{1}{4}\left[-89, -17, 10, -11\right] := v'$

i.e. $\bar{c}_{NB}' = c_{NB}' - v' = (-6, 0, 0, 0) - v'$.

w/ all components positive except the third one, which is negative.

Thus <u>NOT</u> optimal soln.

Hmwk 2.2

See Ex 3.3 of BT.

Let F denote the set of feasible directions $\vec{d}$ at the point $x' = (0,0,1)$ for the set P.

Notice $A = [1 \ 1 \ 1]$. Notice x_3 is basic.

$$[1 \ 1 \ 1] \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

$$\left\{ \begin{array}{l} d_1 + d_2 + d_3 = 0 \quad \star \\ \text{and } d_1 \geq 0, d_2 \geq 0 \end{array} : (d_1, d_2, d_3) \in \mathbb{R}^3 \right\} = F.$$

by Ex 3.3.

Let $c' = (3,2,1)$. Then $c_B' = 1$.

Notice $B = [1]$, $B^{-1} = [1]$., $B^{-1}[A_1 \, A_2] = [1 \ 1]$

So $p' = 1$.

Now $p'[A_1 \, A_2] = [1 \ 1] = v'$., $\vec{u}_1 = 1$, $\vec{u}_2 = 1$

i.e. $\bar{c}_{NB}' = (3,2) - (1,1) = (2,1)$

j-th basic directions:

$j = 1$,	$j = 2$
$\vec{d}_B = -1$	$\vec{d}_B = -1$
$\vec{d} = (1,0,-1)$	$\vec{d} = (0,1,-1)$