Apm236h1f
E. Mazzeo
Quiz #1
Friday, October 9th, 2015
Total: 24 marks

Last Name: **First Name:** **Student #:**

3 marks
1. Convert the following linear programming problem (l.p.p.) to matrix form Ax>=b.
Let x=(r, s, u, v).

Minimize $2r - s + 4u$

subject to
$$\begin{aligned} r + s + v &\le 3 \quad (1)\\ 3s - u &= 6 \quad (2)\\ u + v &\ge 4\\ r &\ge 1\\ u &\le 1 \quad (3)\end{aligned}$$

1: $r+s+v \le 3 \iff -r-s-v \ge -3$

2: $3s-u=6 \iff \begin{cases} 3s-u \le 6 \\ 3s-u \ge 6 \end{cases} \iff \begin{cases} -3s+u \ge 6 \\ 3s-u \ge 6 \end{cases}$

3: $u \le 1 \iff -u \ge -1$

So the constraints are:
$$\begin{aligned} -r-s-v &\ge -3\\ -3s+u &\ge -6\\ 3s-u &\ge 6\\ u+v &\ge 4\\ r &\ge 1\\ -u &\ge -1\end{aligned}$$

Matrix form:
$$\begin{pmatrix} -1 & -1 & 0 & -1 \\ 0 & -3 & 1 & 0 \\ 0 & 3 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} r \\ s \\ u \\ v \end{pmatrix} \ge \begin{pmatrix} -3 \\ -6 \\ 6 \\ 4 \\ 1 \\ -1 \end{pmatrix}$$

6 marks

2. Convert the following linear programming problem (l.p.p.) to "standard (equality) form" Ax=b with x>=0. (a.k.a. "canonical form").

optimize -10 r -12 s -12 u

subject to

r + 2 s + 2 u	<= 10	x_1 slack
-2 r - s -2 u	>= -20	x_2 surplus
2 r + 2 s + u	<= 30	x_3 slack
r	<=0	$\tilde{r} = -r$
s	>=0	
u	>=0	

$r+2s+2u \le 10 \iff \exists\, x_1 \ge 0$ s.t. $r+2s+2u+x_1 = 10$

$-2r-s-2u \ge -20 \iff \exists\, x_2 \ge 0$ s.t. $-2r-s-2u-x_2 = -20$

$2r+2s+u \le 30 \iff \exists\, x_3 \ge 0$ s.t. $2r+2s+u+x_3 = 30$

$r \le 0 \Rightarrow$ we replace r by $\tilde{r} = -r$.

We get:

optimize $10\tilde{r} - 12s - 12u$

s.t.

$$
\begin{aligned}
-\tilde{r} + 2s + 2u + x_1 &= 10 \\
2\tilde{r} - s - 2u - x_2 &= -20 \\
-2\tilde{r} + 2s + u + x_3 &= 30 \\
\tilde{r},\ s,\ u,\ x_1,\ x_2,\ x_3 &\ge 0
\end{aligned}
$$

6 marks

3. Convert the following piecewise linear convex (PLC) problem to a (general) linear programming problem. Explicitly state your new decision variables.

Minimize $\quad \max\{2x+3y, -5x+6y\}$

subject to
$$
\begin{aligned}
x+y &<=4 \\
5x+3y &<=15 \\
x &>=0 \\
y &>=0
\end{aligned}
$$

Let $z=\max\{2x+3y, -5x+6y\}$.

So $\quad z \geq 2x+3y$

$\quad\quad z \geq -5x+6y$

The problem is

$$
\begin{aligned}
\text{Min} \quad & z \\
\text{s.t.} \quad & x+y \leq 4 \\
& 5x+3y \leq 15 \\
& 2x+3y-z \leq 0 \\
& -5x+6y-z \leq 0 \\
& x, y \geq 0
\end{aligned}
$$

The decision variables are x, y and z.

9 marks

4. Consider the linear programming problem.

Minimize $\quad a\,x + b\,y$

subject to

$-x+2y \le 4 \qquad y = 2 + \frac{x}{2} \qquad (0,2),\ (2,3)$

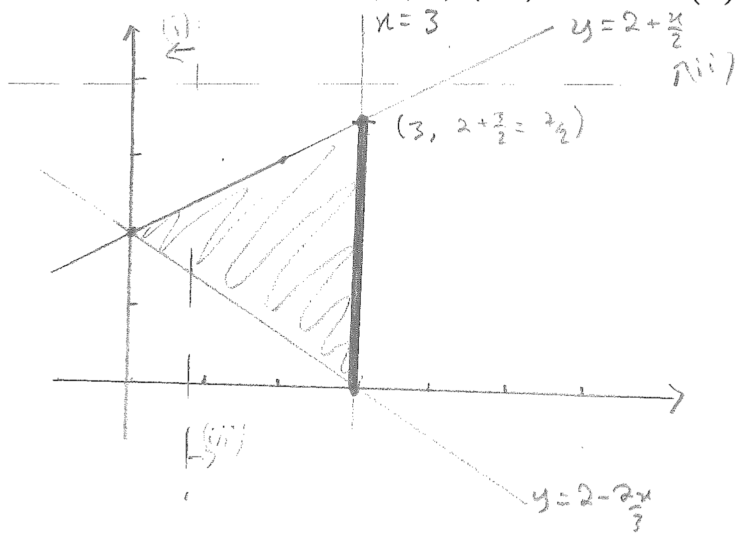
$x \le 3$

$2x+3y \ge 6 \qquad y = 2 - \frac{2x}{3} \qquad (0,2),\ (3,0)$

Part A. Draw the feasibility set.

Part B. Find all optimal solutions (x,y) when:

(i) (a,b)=(1,0) (ii), (a,b)=(0,-1) (iii) (a,b)=(-1,0)



(i) $(a,b)=(1,0)$ ← optimal for $(0,2)$

(ii) $(a,b)=(0,-1)$ optimal for $(3, 7/2)$

(iii) $(a,b)=(-1,0)$ optimal for the line $(3,y)$ $0 \le y \le 7/2$