Apm236h1f
E. Mazzeo

Quiz #2

Friday, October 30th, 2015
Total: 24 marks

Last Name: **First Name:** **Student #:**

3 marks

1. Show using the precise definition of convexity (that is, not geometrically) that the upper halfplane H={(x,y) | y>0} in R^2 is a convex set.

Let $p_1=(x_1,y_1), p_2=(x_2,y_2) \in H$. We need to show that the line between p_1 and p_2 is also in H, i.e.

$$\forall \lambda \in [0,1] \quad \lambda p_1 + (1-\lambda)p_2 \in H.$$

We have, $\lambda p_1 + (1-\lambda)p_2 = (\lambda x_1 + (1-\lambda)x_2, \lambda y_1 + (1-\lambda)y_2)$.

As $p_1, p_2 \in H$, $y_1, y_2 > 0$, so

$$\lambda y_1 + (1-\lambda)y_2 > 0.$$

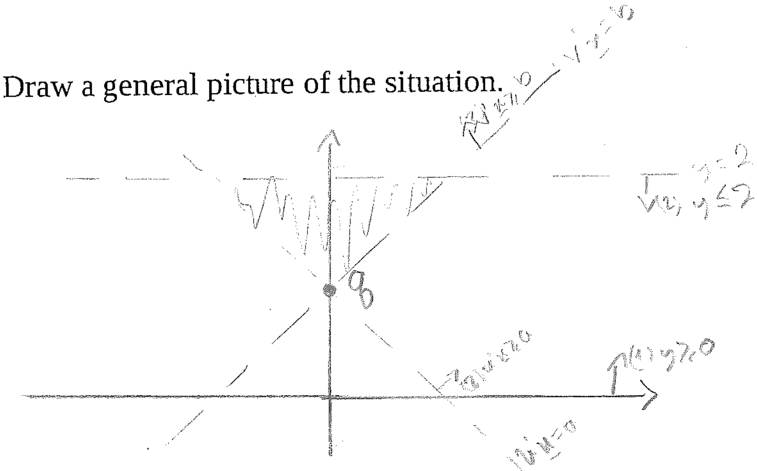
Thus, $\lambda p_1 + (1-\lambda)p_2 \in H$.

5 marks

2. Let **u,v** be given **vectors** in R2 and a,b given real numbers. Let <u>**x**</u>=(x,y) be a vector. Suppose the two hyperplanes (lines) **u**' <u>**x**</u>=a and **v**' <u>**x**</u>=b meet at the point **q**=(0,1) and the lines are <u>perpendicular</u> to each other. Let P be the polyhedron defined by the following contraints:

(1) y>=0, (2) **u**' <u>**x**</u>>=a (3) **v**' <u>**x**</u>>=b (4) y<=2

(a) Draw a general picture of the situation.

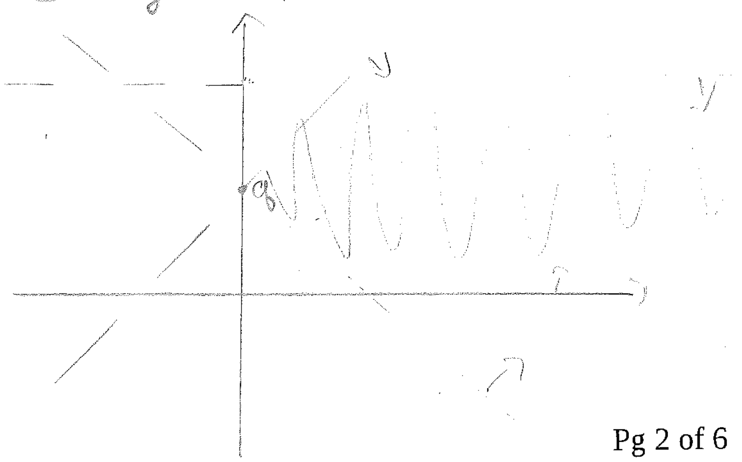


(b) Give an example of two vectors **u, v** so that the P is <u>bounded</u>.

As above $u=(1,1)$ $v=(-1,1)$.

(c) Give an example of two vectors **u, v** so that the P is <u>unbounded</u>.

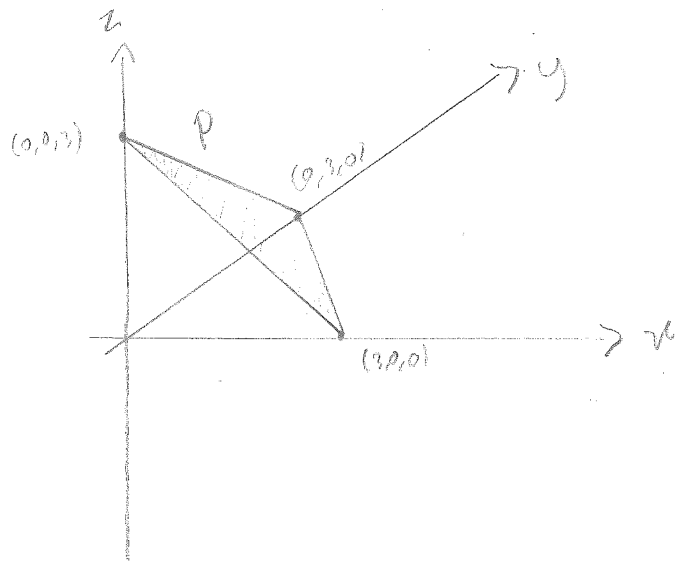
Change v from (b) to $(1,-1)$.



4 ~~6~~ **marks**

3. Consider the polyhedra P defined by the following constraints: x+y+z=3, x>=0, y>=0, z>=0

(a) Draw an accurate picture of P labelling its corner points.



(b) List the constraints that are active at the vector a=(3,0,0).

$$\begin{cases} x+y+z=3 \\ y=0 \\ z=0 \end{cases}$$

(c) List the constraints that are active at the vector v=(1,1,1).

$x+y+z=3$

10 marks

4. Consider the following l.p.p. in standard (matrix equality) form with $\mathbf{x} = (r, s, u, v, w) >= 0$.

$$
\begin{array}{llllll}
(1) & & u & & & = 2 \\
(2) & 2s & & + v & & = 4 \\
(3) & -6r + 3s & & & + w & = 6
\end{array}
$$

(a) Write it in standard (matrix equality) form A**x**=**b**.

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 \\ -6 & 3 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r \\ s \\ u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$$

(b) Find a basic <u>feasible (non-degenerate)</u> solution **x** and state the corresponding basic variables.

$x = (0 \quad 0 \quad 2 \quad 4 \quad 6)$

Basic variables: u, v, w

(c) Find a basic <u>infeasible</u> solution **y** and state the corresponding basic variables.

$y = (-1 \quad 0 \quad 2 \quad 4 \quad 0)$

Infeasible because $r \ngeq 0$.

Basic variables: r, u, v

(d) Find a basic feasible degenerate solution $\mathbf{z}$ and state the corresponding basic variables.

$z = (0 \quad 2 \quad 2 \quad 0 \quad 0)$

Basic variables: s, u and r or v or w.