Deadline Wednesday, November 7.

APM 346 (2012) Home Assignment 6

Some of the problems in this assignment could be solved based on the other problems and properties of Fourier transform (see Lecture 18) and such solutions are much shorter than from the scratch; seeing and exploiting connections is a plus.

Problem 1 Let $\alpha > 0$. Find Fourier transforms of

(a) $e^{-\alpha|x|}$;

(b) $e^{-\alpha|x|}\cos(\beta x)$, $e^{-\alpha|x|}\sin(\beta x)$ with $\beta > 0$;

(c) $xe^{-\alpha|x|}$ with $\beta > 0$;

(d) $xe^{-\alpha|x|}\cos(\beta x)$, $xe^{-\alpha|x|}\sin(\beta x)$ with $\beta > 0$.

Problem 2 Let $\alpha > 0$. Find Fourier transforms of

(a) $(x^2+\alpha^2)^{-1}$;

(b) $x(x^2+\alpha^2)^{-1}$;

(c) $(x^2+\alpha^2)^{-1}cos(\beta x)$, $(x^2+\alpha^2)^{-1}\sin(\beta x)$;

(d) $x(x^2+\alpha^2)^{-1}cos(\beta x)$, $x(x^2+\alpha^2)^{-1}\sin(\beta x)$.

Problem 3 Let $\alpha > 0$. Based on Fourier transform of $e^{-\alpha x^2/2}$ find Fourier transforms of

(a) $e^{-\alpha x^2/2}\cos(\beta x)$, $e^{-\alpha x^2/2}\sin(\beta x)$;

(b) $xe^{-\alpha x^2/2}\cos(\beta x)$, $xe^{-\alpha x^2/2}\sin(\beta x)$.

Problem 4 Find Fourier transforms of

(a) $f(x) = \begin{cases} 1 & |x| \le a, \\ \\ 0 & |x| \ge a; \end{cases}$

(b) $f(x) = \begin{cases} x & |x| \le a, \\ \\ 0 & |x| \ge a; \end{cases}$

(c) Using (a) calculate $\int_{-\infty}^{\infty} \frac{\sin(x)}{x}\, dx$.